Embedded Systems
2012/13

Basilica di Santa Maria di Collemaggio, 1287, L'Aquila

Lecture 9
Networked control systems: symbolic models and integrated symbolic control design

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Networked control systems

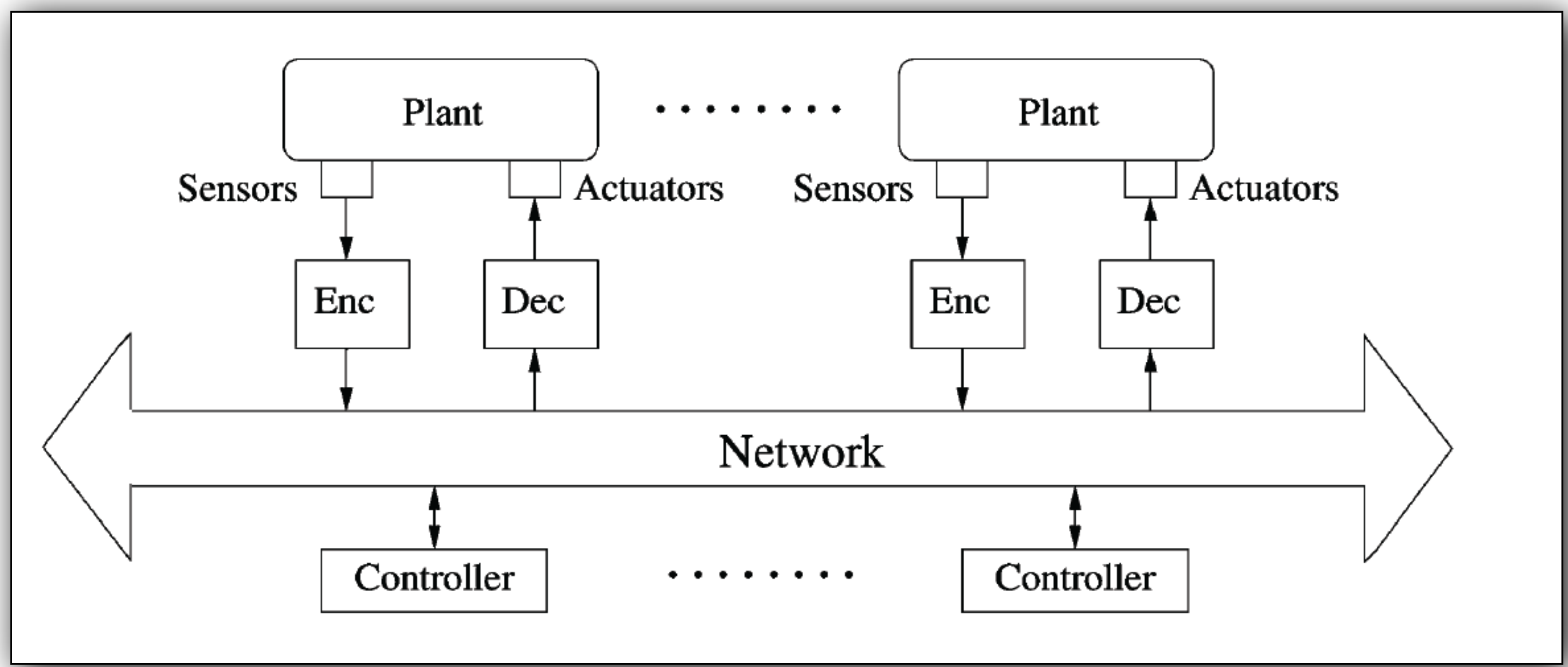
- Networked Control Systems (NCS) are spatially distributed systems where the communication among plants, sensors, actuators and controllers occurs in a shared communication network
- At present, most of the results concerning NCS focus on stability and stabilizability problems
- Results available in the literature vary depending on the class of systems considered (linear vs. nonlinear), controllers synthesized (continuous vs. digital), and assumptions on the network non-idealities

Outline

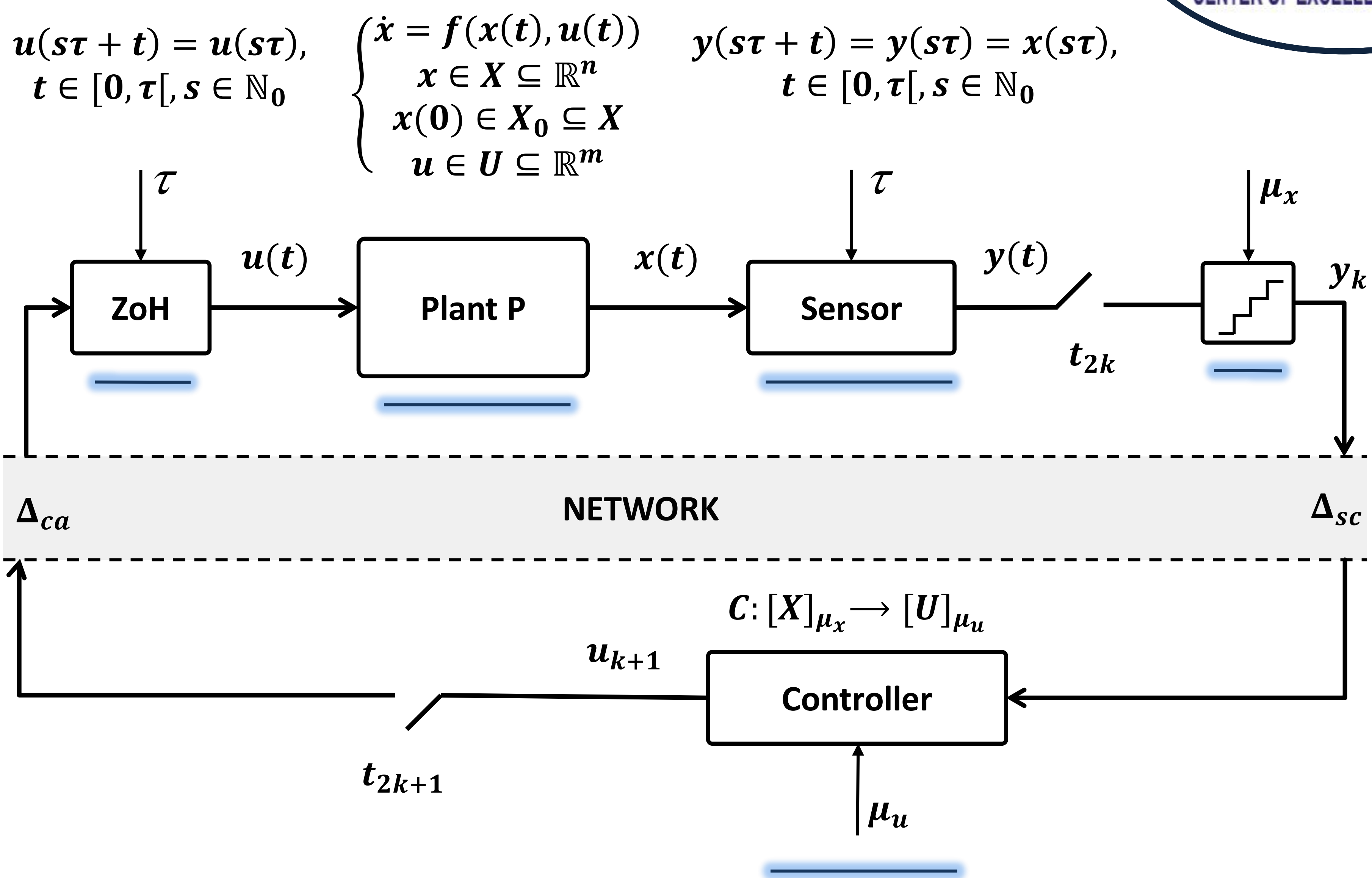
Symbolic Control Design of Nonlinear Networked Control Systems

- Mathematical model of nonlinear NCS
- Symbolic models for stable and unstable NCS
- Symbolic control design of NCS
- Efficient control design algorithms

Symbolic Control Design of Nonlinear Networked Control Systems



Networked control systems: Our model



Networked control systems: Our model

Network non-idealities: quantization, packet drops, variable delays *(e.g. [Andersson, IEEE-CDC-05], [Antsaklis, IEEE-TAC-04], [Heemels, IEEE-TAC-10], [Hespanha, Proc. IEEE-07], [Murray, SMTNS-06])*

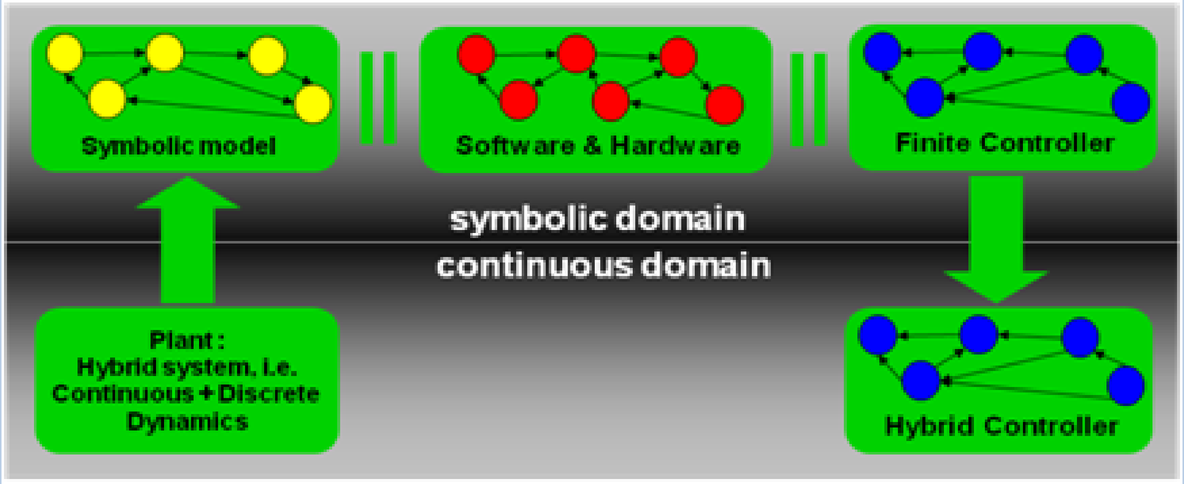
Network and computing non-idealities in our model:

- Quantization errors
- Bounded time-varying network access times
- Bounded time-varying communication delays induced by the network
- Bounded time-varying computation time of computing units
- Limited bandwidth
- Bounded packet losses

Correct-by-design controller synthesis

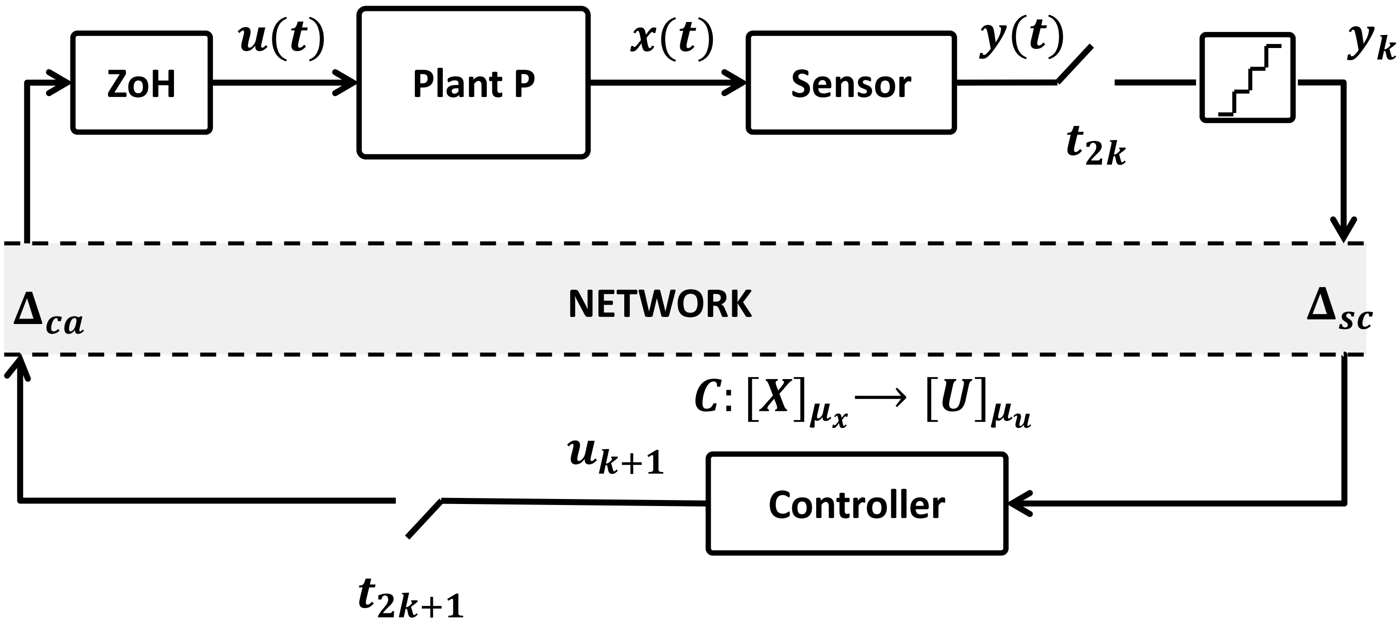
… a three-step process:

1. Construct the finite/symbolic model T of the plant system Σ
2. Design a finite/symbolic controller C that solves the specification S for T
3. Refine the controller C to obtain controller C’ for Σ



Dealing with heterogeneity

Nonlinear Networked control systems as LTSs



t	0	τ	2τ	3τ	4τ	5τ	6τ	7τ	8τ	9τ	…
u	0	0	0	u_1	u_1	u_1	u_1	u_1	u_1	u_2	…
x	x(0)	$x(\tau)$	$x(2\tau)$	$x(3\tau)$	$x(4\tau)$	$x(5\tau)$	$x(6\tau)$	$x(7\tau)$	$x(8\tau)$	$x(9\tau)$	…
	$N_1 = 4$				$N_2 = 6$						…

Dealing with heterogeneity

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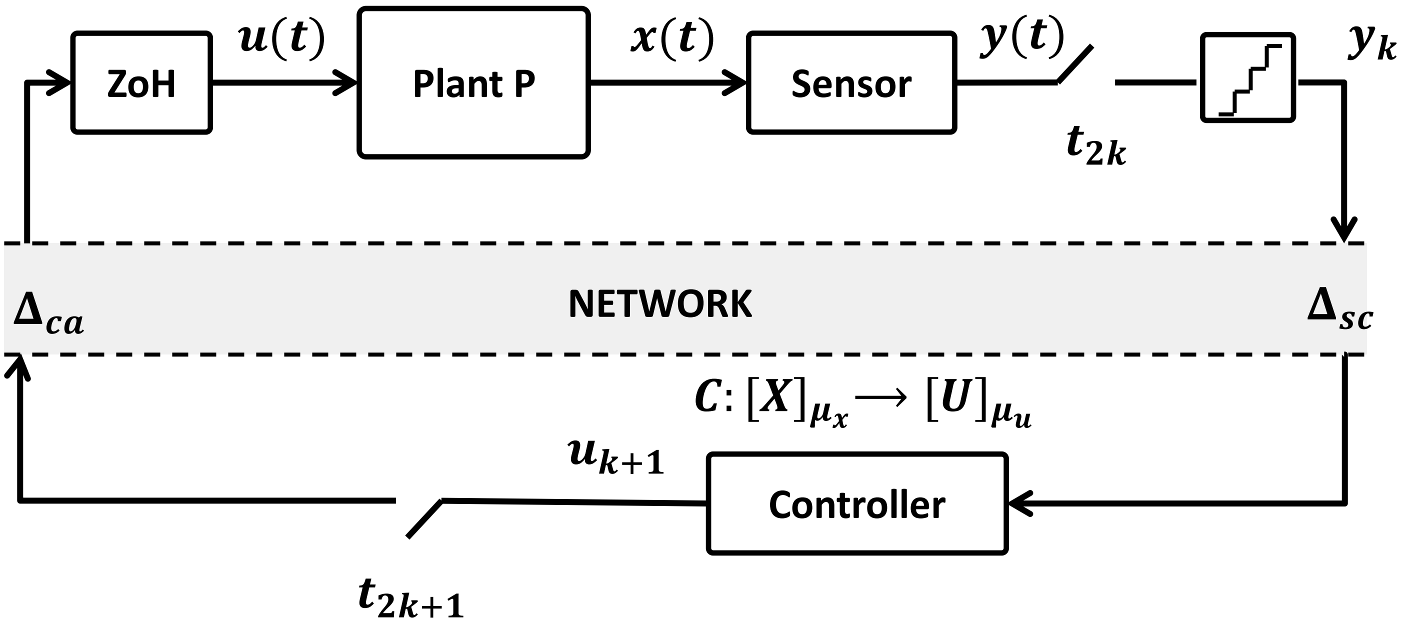

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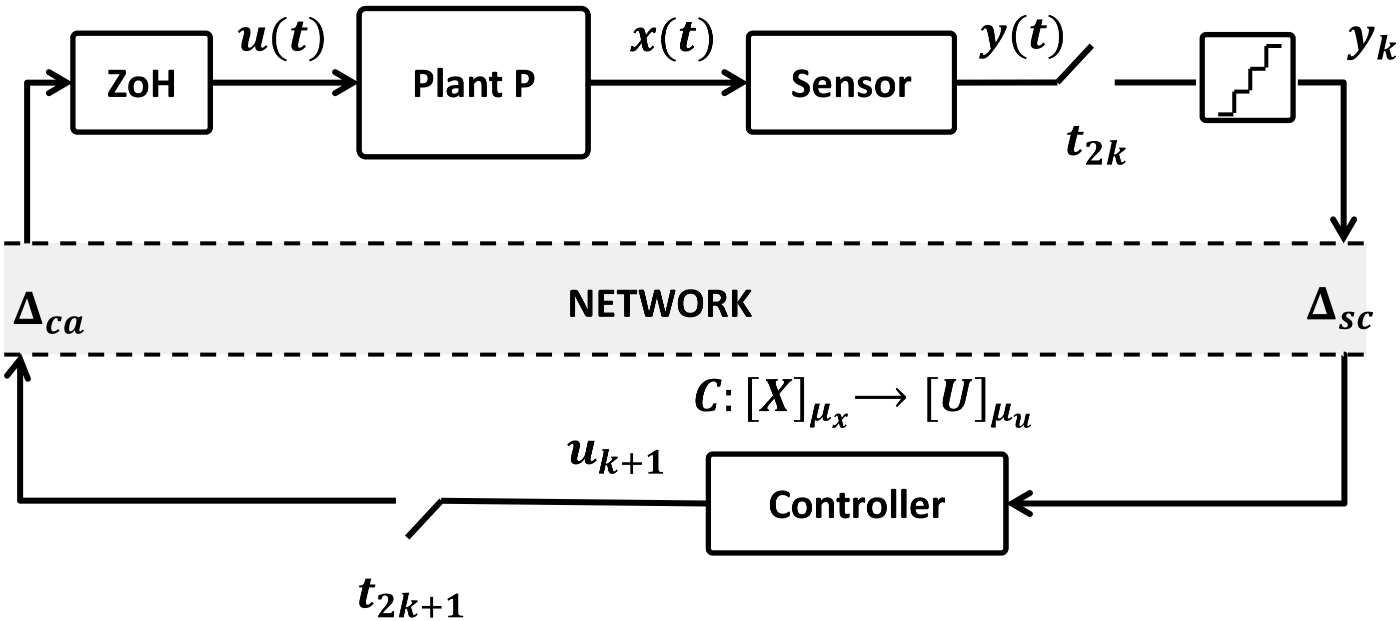

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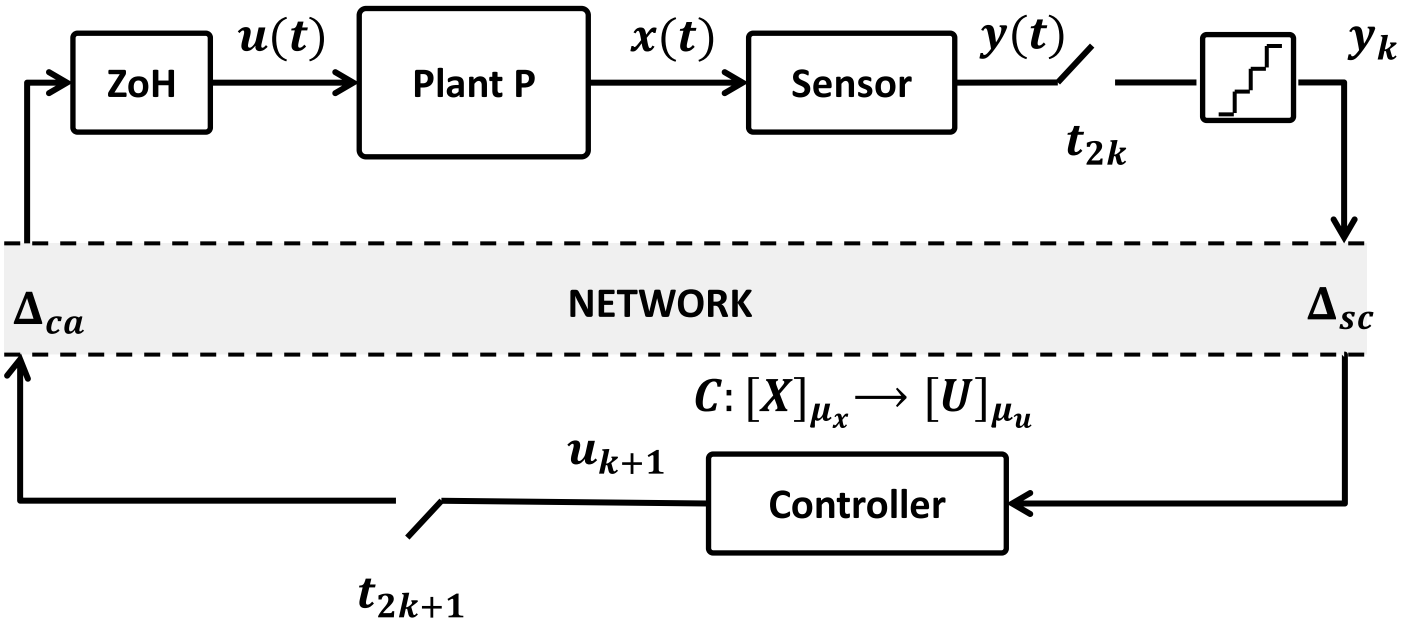

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Dealing with heterogeneity

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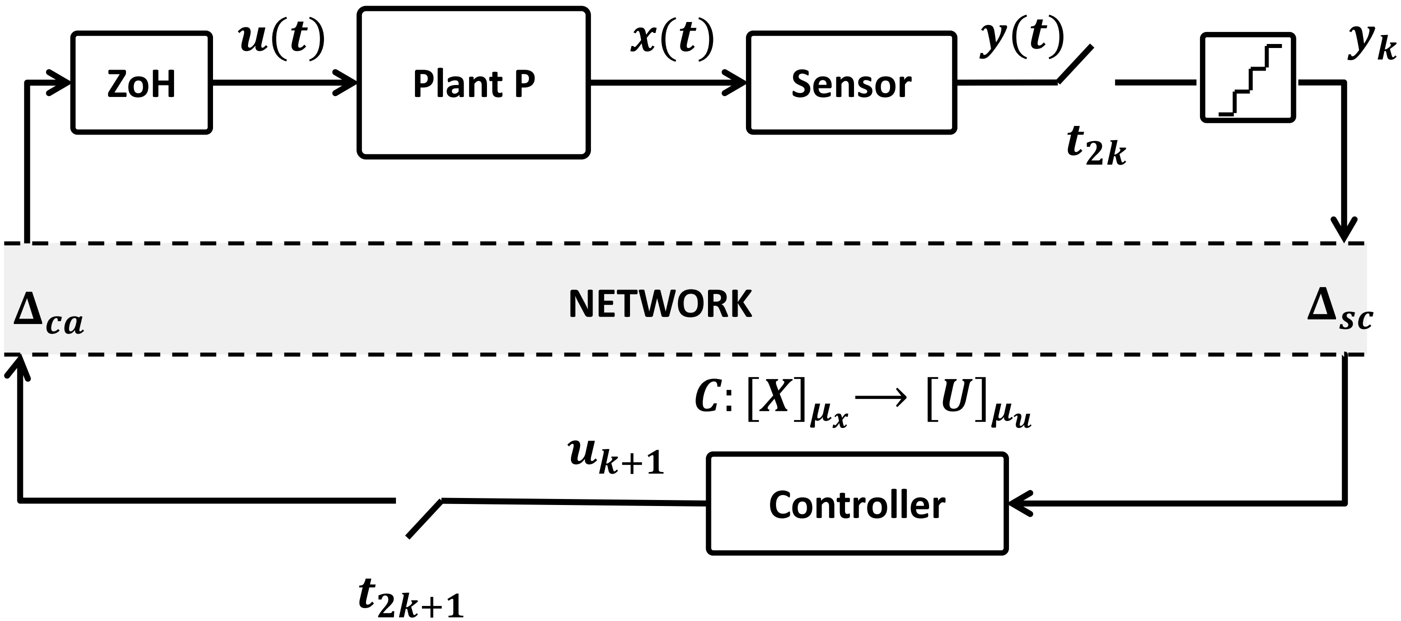

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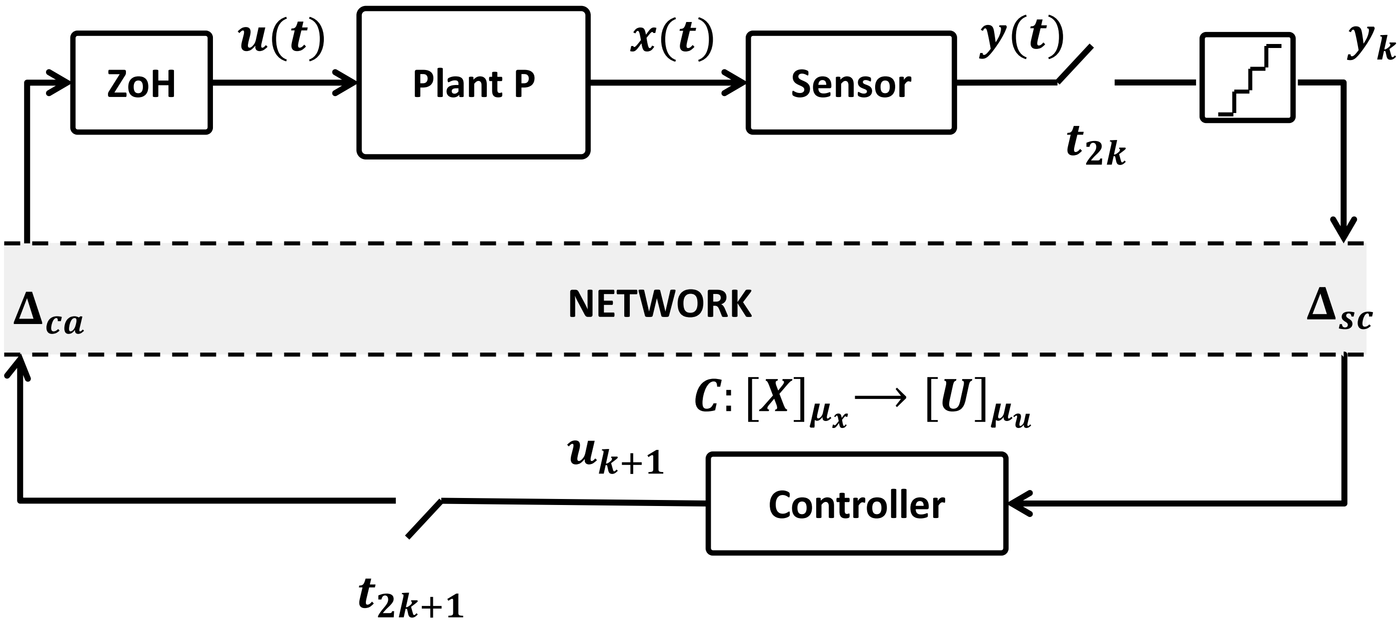
Dealing with heterogeneity

Nonlinear Networked control systems as LTSs



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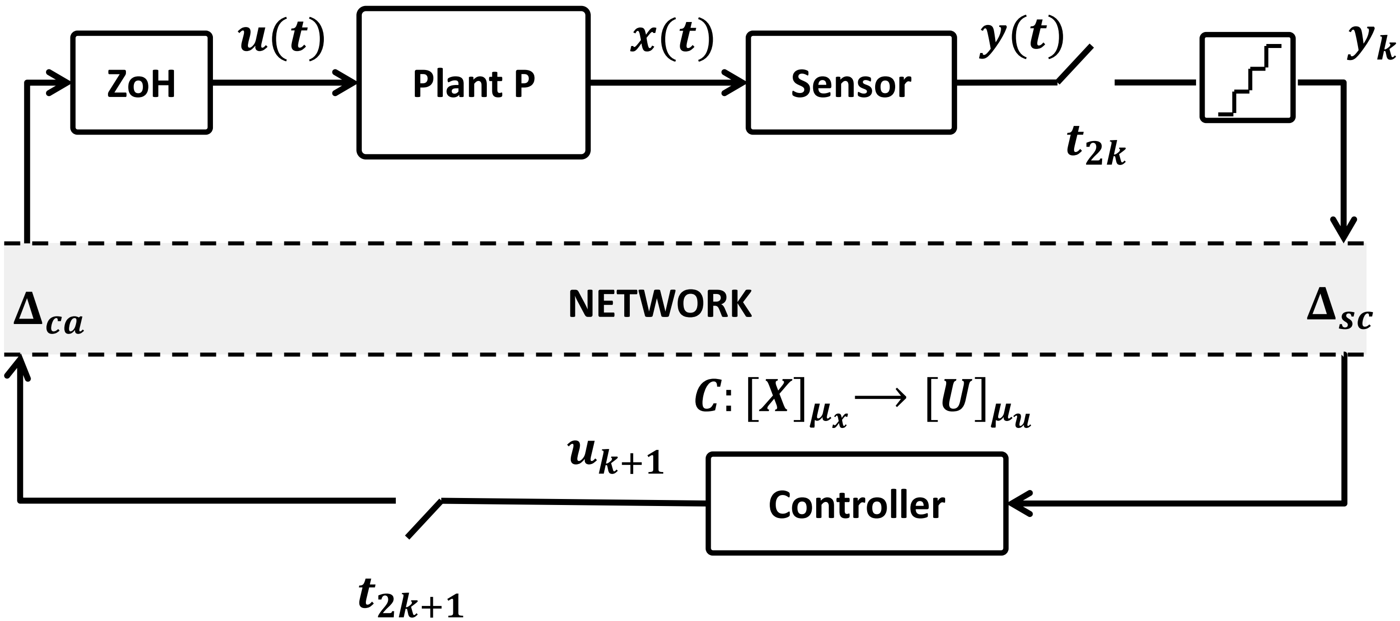

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Dealing with heterogeneity

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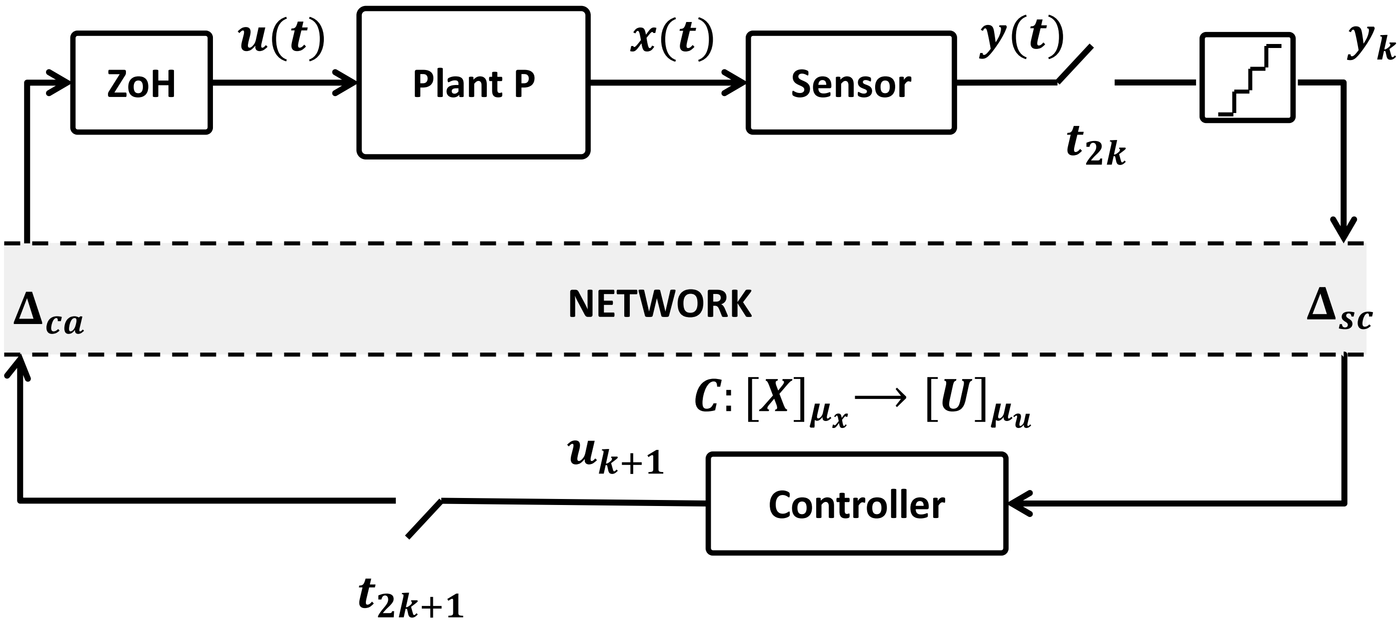

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Dealing with heterogeneity

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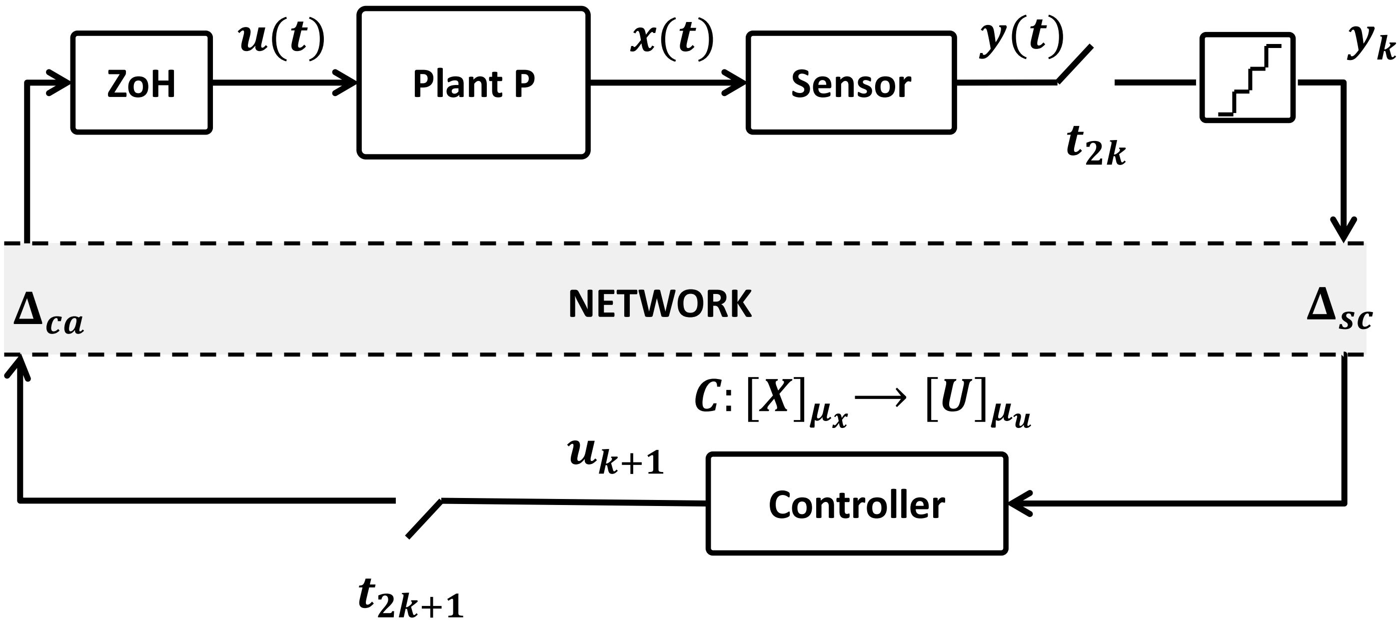

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Dealing with heterogeneity

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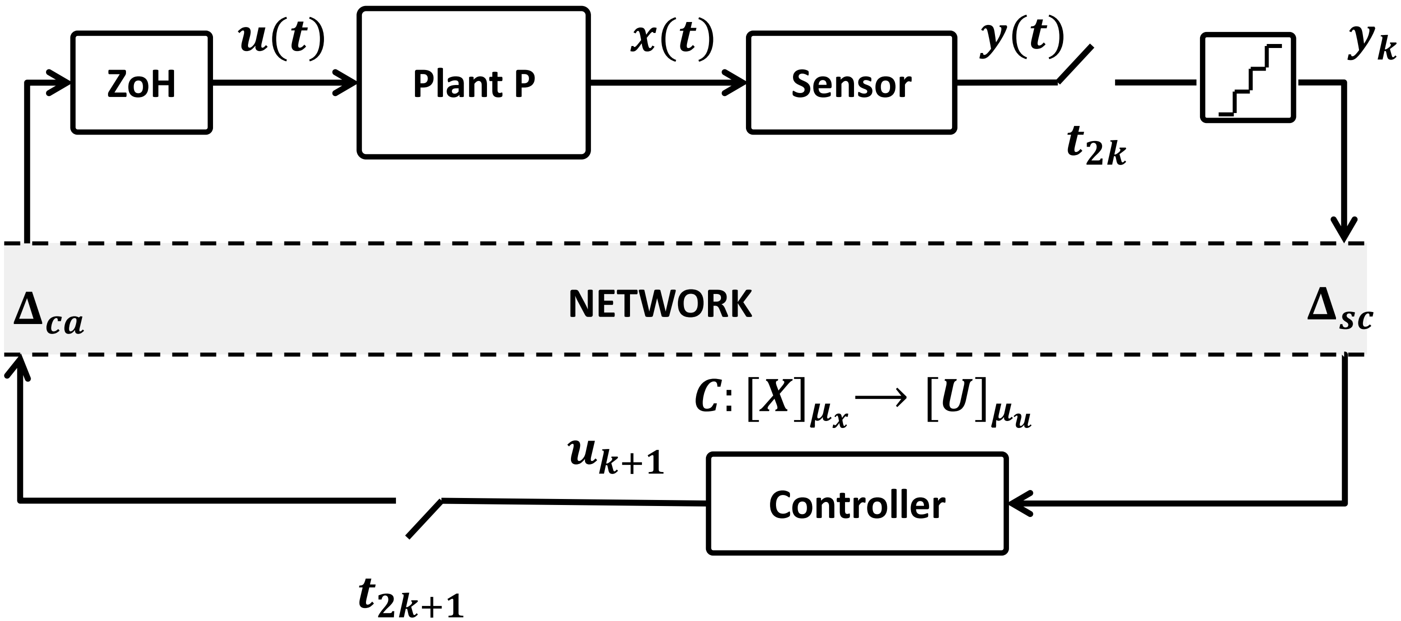

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x	x(0)	x(τ)	x(2τ)	x(3τ)	x(4τ)	x(5τ)	x(6τ)	x(7τ)	x(8τ)	x(9τ)	…
	$N_1 = 4$				$N_2 = 6$						…

Dealing with heterogeneity

Nonlinear Networked control systems as LTSs

$$(x(0),x(\tau),x(2\tau),x(3\tau)) \xrightarrow{u_1} (x(4\tau),x(5\tau),x(6\tau),x(7\tau),x(8\tau),x(9\tau))$$

t	0	τ	2τ	3τ	4τ	5τ	6τ	7τ	8τ	9τ	…
u	0	0	0	u_1	u_1	u_1	u_1	u_1	u_1	u_2	…
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	$N_1 = 4$				$N_2 = 6$						…

Dealing with heterogeneity

Nonlinear Networked control systems as LTSs

$$(x(0),x(\tau),x(2\tau),x(3\tau)) \xrightarrow{u_2} (x(4\tau),x(5\tau),x(6\tau),x(7\tau),x(8\tau),x(9\tau)) \quad (N_2=6)$$

$$(x(0),x(\tau),x(2\tau),x(3\tau)) \xrightarrow{u_2} (x(4\tau),x(5\tau),x(6\tau),x(7\tau)) \quad (N_2=4)$$

$$(x(0),x(\tau),x(2\tau),x(3\tau)) \xrightarrow{u_2} (x(4\tau),x(5\tau),x(6\tau),x(7\tau),x(8\tau)) \quad (N_2=5)$$

t	0	τ	2τ	3τ	4τ	5τ	6τ	7τ	8τ	9τ	…
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x	$x(0)$	$x(\tau)$	$x(2\tau)$	$x(3\tau)$	$x(4\tau)$	$x(5\tau)$	$x(6\tau)$	$x(7\tau)$	$x(8\tau)$	$x(9\tau)$	…
	$N_1 = 4$				$N_2 = 6$						…

Dealing with heterogeneity

Given a NCS Σ define the LTS

$T(\Sigma) = (Q_\tau, Q_{0,\tau}, L_\tau, \longrightarrow_\tau, O_\tau, H_\tau)$ where:

- $Q_\tau \subseteq Q_0 \cup Q_e$ where $Q_e := \bigcup_{N=N_{min}}^{N_{max}} Q^N$ and for any $q = (x_1, x_2, \dots, x_N) \in Q^N, x_{i+1} = \boldsymbol{x}(\tau, x_i, u^-)$, $i \in [1; N-2]$, and $x_N = \boldsymbol{x}(\tau, x_{N-1}, u^+)$ for some control inputs u^-, u^+

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- $Q_{0,\tau} = Q_0$

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- $L_\tau = [U]_{\mu_U}$

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- $q^1 \xrightarrow{\text{u}}_\tau \; q^2$ where, for some $N_1, N_2 \in [N_{min}; N_{max}]$

Dealing with heterogeneity

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- $q^1 \xrightarrow{\mathrm{u}}_\tau \; q^2$ where, for some $N_1, N_2 \in [N_{min}; N_{max}]$
  $x_{i+1}^1 = \boldsymbol{x}(\tau, x_i^1, u_1^-), i \in [1; N_1 - 2]$
  $x_N^1 = \boldsymbol{x}(\tau, x_{N_1-1}^1, u_1^+)$

Dealing with heterogeneity

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  $x_{i+1}^2 = \boldsymbol{x}(\tau, x_i^2, u_2^-), i \in [1; N_2 - 2]$
  $x_N^2 = \boldsymbol{x}\left(\tau, x_{N_2-1}^2, u_2^+\right)$

Dealing with heterogeneity

Given a NCS Σ define the LTS

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- $Q_{0,\tau} = Q_0$
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- $q^1 \xrightarrow{\mathrm{u}}_\tau \; q^2$ where, for some $N_1, N_2 \in [N_{min}; N_{max}]$

$$
\begin{aligned}
x_{i+1}^1 &= \boldsymbol{x}(\tau, x_i^1, u_1^-),\ i \in [1; N_1 - 2] \\
x_N^1 &= \boldsymbol{x}(\tau, x_{N_1-1}^1, u_1^+) \\
x_{i+1}^2 &= \boldsymbol{x}(\tau, x_i^2, u_2^-),\ i \in [1; N_2 - 2] \\
x_N^2 &= \boldsymbol{x}\left(\tau, x_{N_2-1}^2, u_2^+\right) \\
u_2^- &= u_1^+ \\
u_2^+ &= u \\
x_1^2 &= \boldsymbol{x}(\tau, x_{N_1}^1, u_2^-)
\end{aligned}
$$

Dealing with heterogeneity

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  $x_N^2 = \boldsymbol{x}\left(\tau, x_{N_2-1}^2, u_2^+\right)$
  $u_2^- = u_1^+$
  $u_2^+ = u$
  $x_1^2 = \boldsymbol{x}(\tau, x_{N_1}^1, u_2^-)$
- $O_\tau = X_\tau$
- H_τ is the identity function

Dealing with heterogeneity

THEOREM 5.1. *Given the NCS Σ and the system $S(\Sigma)$ the following properties hold:*

- *for any trajectory $\mathbf{x}(., x_0, u) \in Traj(\Sigma)$ of Σ, there exists a state run*

$$x^0 \xrightarrow{u_1} x^1 \xrightarrow{u_2} \ldots, \tag{11}$$

of $S(\Sigma)$ with $x^i = (x^i_1, x^i_2, \ldots, x^i_{N_i})$ such that $x^0 = x_0$ and the sequence of states

$$x^0 \quad , \quad \underbrace{x^1_1, \ldots, x^1_{N_0+1}}_{x^1} \quad , \quad \underbrace{x^2_1, \ldots, x^2_{N_1}}_{x^2} \quad , \quad \ldots \tag{12}$$

obtained by concatenating each component of the vectors x^i, coincides with the sequence of sensor measurements

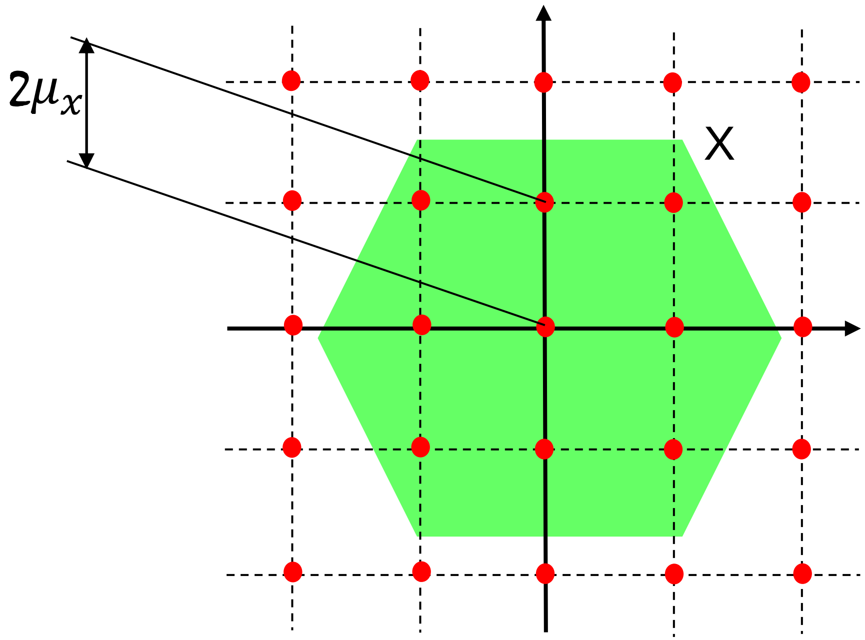
$$\begin{aligned} &y(0), y(\tau), \ldots, y((N_0+1)\tau), y((N_0+2)\tau), \ldots, \\ &y((N_0+N_1+1)\tau), \ldots \end{aligned} \tag{13}$$

in the NCS Σ;

- *for any state run (11) of $S(\Sigma)$, there exists a trajectory $\mathbf{x}(., x_0, u) \in Traj(\Sigma)$ of Σ such that the sequence of states in (12) coincides with the sequence (13) of sensor measurements in the NCS Σ.*

Symbolic models for NCS

T(Σ) collects all the information of the NCS Σ available at the sensor, but it is not a symbolic model. We therefore propose a symbolic model by quantizing the state space X of the plant P



Given x ∈ X let $[x]_{\mu_X} \in [X]_{\mu_X}$ be such that $\| x - [x]_{\mu_X} \| \leq \mu_X$

Symbolic models for NCS

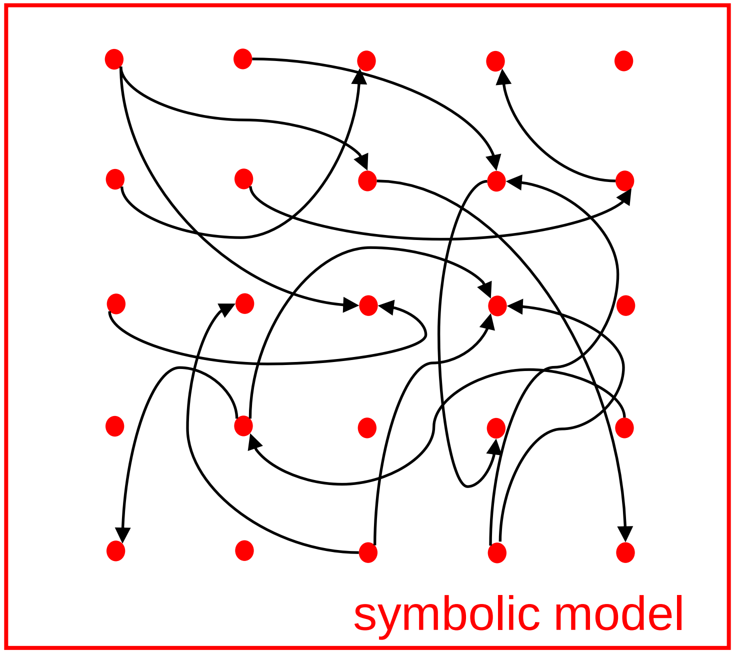
Define the system $T^*(\Sigma) = (Q_*, Q_{0,*}, L_*, \longrightarrow_*, O_*, H_*)$ where:

- $Q_* \subseteq [Q_0 \cup Q_e]_{\mu_x}$ s.t. for any $q^* = (x_1^*, x_2^*, \dots, x_N^*) \in Q_*$, $x_{i+1}^* = [\boldsymbol{x}(\tau, x_i^*, u_*^-)]_{\mu_x}$, $i \in [1; N-2]$, and $x_N^* = [\boldsymbol{x}(\tau, x_{N-1}^*, u_*^+)]_{\mu_x}$ for some u_*^-, u_*^+
- $Q_{0,*} = [X_0]_{\mu_x}$
- $L_* = [U]_{\mu_u}$
- $q^1 \longrightarrow_*^{u_*} q^2$ where, for some N_1, N_2

$$x_{i+1}^1 = [\boldsymbol{x}(\tau, x_i^1, u_1^-)]_{\mu_x}, \; i \in [1; N-2]$$
$$x_N^1 = [\boldsymbol{x}(\tau, x_{N_1-1}^1, u_1^+)]_{\mu_x}$$
$$x_{i+1}^2 = [\boldsymbol{x}(\tau, x_i^2, u_2^-)]_{\mu_x}, \; i \in [1; N-2]$$
$$x_N^2 = [\boldsymbol{x}(\tau, x_{N_2-1}^2, u_2^+)]_{\mu_x}$$
$$u_2^- = u_1^+$$
$$u_2^+ = u_*$$
$$x_1^2 = [\boldsymbol{x}(\tau, x_{N_1}^1, u_2^-)]_{\mu_x}$$

- $O_* = X_\tau$
- H_* is the identity function

By construction $S_*(\Sigma)$ is a symbolic model!



Symbolic models for NCS

Def [Angeli, IEEE-TAC-2002]

Given a nonlinear control system $\dot{x} = f(x, u)$, a smooth function

$$V: \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}_0^+$$

is said to be a δ-GAS Lyapunov function for P if there exist $\lambda \in \mathbb{R}^+$ and K_∞ functions α_1, α_2 such that, for any $x_1, x_2 \in \mathbb{R}^n$ and any $u \in U$

1) $\alpha_1(\| x_1 - x_2 \|) \leq V(x_1,x_2) \leq \alpha_2 (\| x_1 - x_2 \|)$;

2) $\frac{\partial V}{\partial x_1} f(x_1, u) + \frac{\partial V}{\partial x_2} f(x_2, u) \leq -\lambda V(x_1,x_2)$.

Theorem [Angeli, IEEE-TAC-2002]

A nonlinear control system $\dot{x} = f(x, u)$ is δ-GAS if it admits a δ-GAS Lyapunov function

Symbolic models for NCS

Theorem 1 [HSCC-2012]

Consider the NCS Σ and suppose that the plant nonlinear control system P enjoys the following properties:

1. There exists a δ-GAS Lyapunov function for Σ, hence there exists $\lambda \in \mathbb{R}^+$ s.t. for any $x_1, x_2 \in X$, and any $u \in U$

$$\frac{\partial V}{\partial x_1} f(x_1, u) + \frac{\partial V}{\partial x_2} f(x_2, u) \leq -\lambda V(x_1, x_2).$$

2. There exists a K_∞ function γ such that $V(x, x') \leq V(x, x'') + \gamma(\| x' - x'' \|)$ for every $x, x', x'' \in X$.

Then for any desired precision $\varepsilon > 0$, any sampling time $\tau > 0$, and any state quantization $\mu_x > 0$ such that

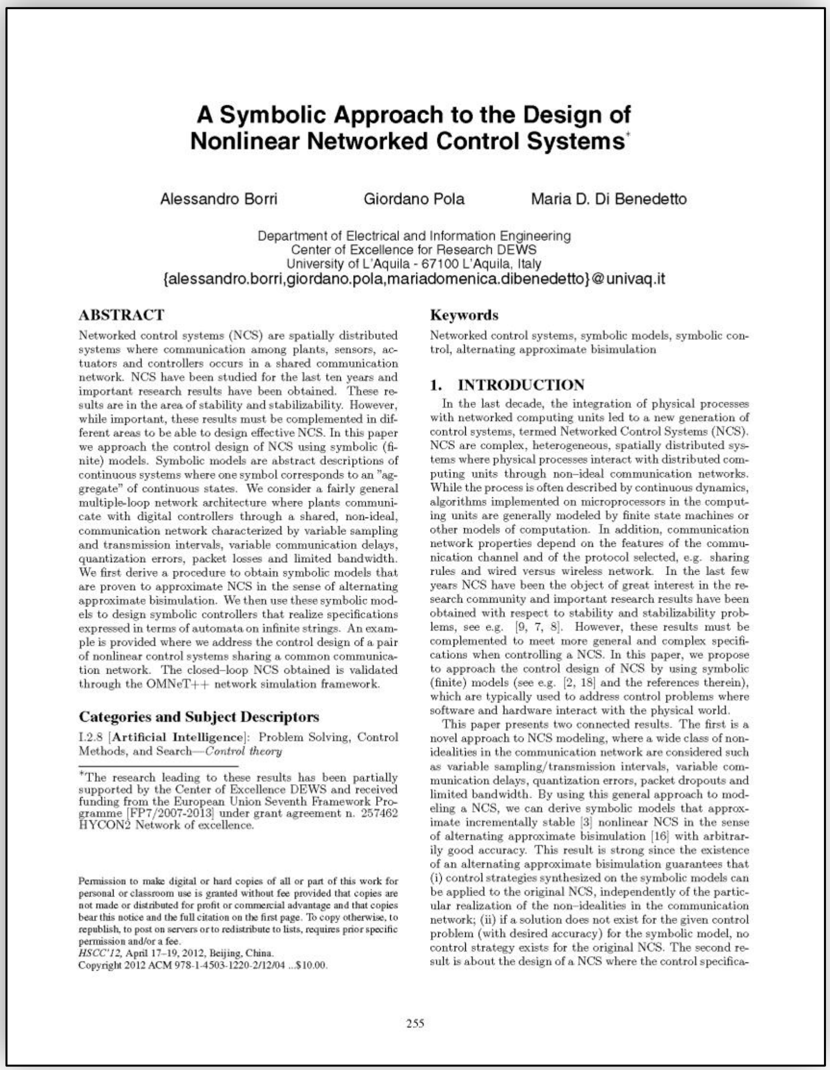
$$\mu_x \leq min\left\{\gamma^{-1}\left(\left(1 - e^{-\lambda\tau}\right)\underline{\alpha}(\varepsilon)\right), \overline{\alpha}^{-1}(\underline{\alpha}(\varepsilon)), \hat{\mu}_X\right\}$$

systems $T(\Sigma)$ and $T^*(\Sigma)$ are ε-alternating bisimilar

Symbolic models

Theorem [HSCC-2012]

For any δ-GAS nonlinear NCS Σ with compact state and input spaces, for any precision ε, there exists a symbolic transition system $T^*(\Sigma)$ that is an ε-alternating approximate bisimulation of Σ and that can be effectively computed



A Symbolic Approach to the Design of Nonlinear Networked Control Systems*

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ABSTRACT

Networked control systems (NCS) are spatially distributed systems where communication among plants, sensors, actuators and controllers occurs in a shared communication network. NCS have been studied for the last ten years and important research results have been obtained. These results are in the area of stability and stabilizability. However, while important, these results must be complemented in different areas to be able to design effective NCS. In this paper we approach the control design of NCS using symbolic (finite) models. Symbolic models are abstract descriptions of continuous systems where one symbol corresponds to an "aggregate" of continuous states. We consider a fairly general multiple-loop network architecture where plants communicate with digital controllers through a shared, non-ideal, communication network characterized by variable sampling and transmission intervals, variable communication delays, quantization errors, packet losses and limited bandwidth. We first derive a procedure to obtain symbolic models that are proven to approximate NCS in the sense of alternating approximate bisimulation. We then use these symbolic models to design symbolic controllers that realize specifications expressed in terms of automata on infinite strings. An example is provided where we address the control design of a pair of nonlinear control systems sharing a common communication network. The closed–loop NCS obtained is validated through the OMNeT++ network simulation framework.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Control theory*

*The research leading to these results has been partially supported by the Center of Excellence DEWS and received funding from the European Union Seventh Framework Programme [FP7/2007-2013] under grant agreement n. 257462 HYCON2 Network of excellence.

Keywords

Networked control systems, symbolic models, symbolic control, alternating approximate bisimulation

1. INTRODUCTION

In the last decade, the integration of physical processes with networked computing units led to a new generation of control systems, termed Networked Control Systems (NCS). NCS are complex, heterogeneous, spatially distributed systems where physical processes interact with distributed computing units through non–ideal communication networks. While the process is often described by continuous dynamics, algorithms implemented on microprocessors in the computing units are generally modeled by finite state machines or other models of computation. In addition, communication network properties depend on the features of the communication channel and of the protocol selected, e.g. sharing rules and wired versus wireless network. In the last few years NCS have been the object of great interest in the research community and important research results have been obtained with respect to stability and stabilizability problems, see e.g. [9, 7, 8]. However, these results must be complemented to meet more general and complex specifications when controlling a NCS. In this paper, we propose to approach the control design of NCS by using symbolic (finite) models (see e.g. [2, 18] and the references therein), which are typically used to address control problems where software and hardware interact with the physical world.

This paper presents two connected results. The first is a novel approach to NCS modeling, where a wide class of non-idealities in the communication network are considered such as variable sampling/transmission intervals, variable communication delays, quantization errors, packet dropouts and limited bandwidth. By using this general approach to modeling a NCS, we can derive symbolic models that approximate incrementally stable [3] nonlinear NCS in the sense of alternating approximate bisimulation [16] with arbitrarily good accuracy. This result is strong since the existence of an alternating approximate bisimulation guarantees that (i) control strategies synthesized on the symbolic models can be applied to the original NCS, independently of the particular realization of the non–idealities in the communication network; (ii) if a solution does not exist for the given control problem (with desired accuracy) for the symbolic model, no control strategy exists for the original NCS. The second result is about the design of a NCS where the control specifica-

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Incremental forward completeness

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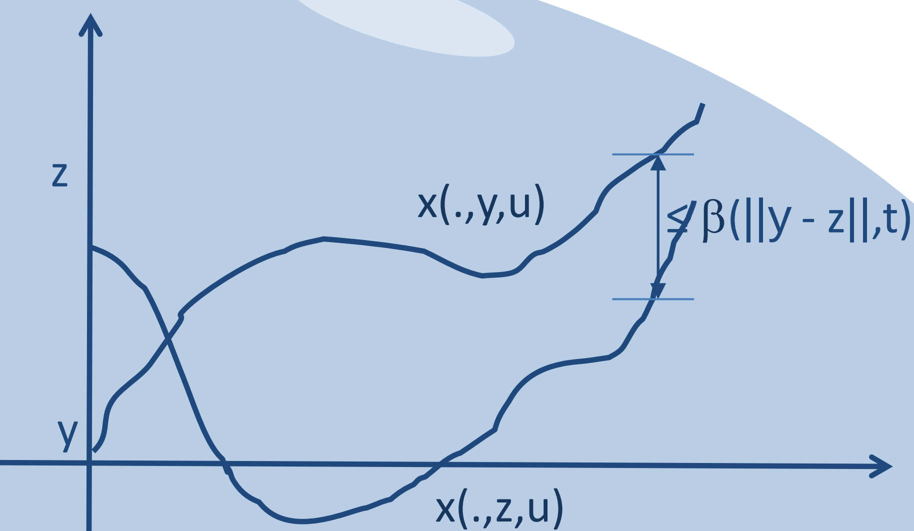
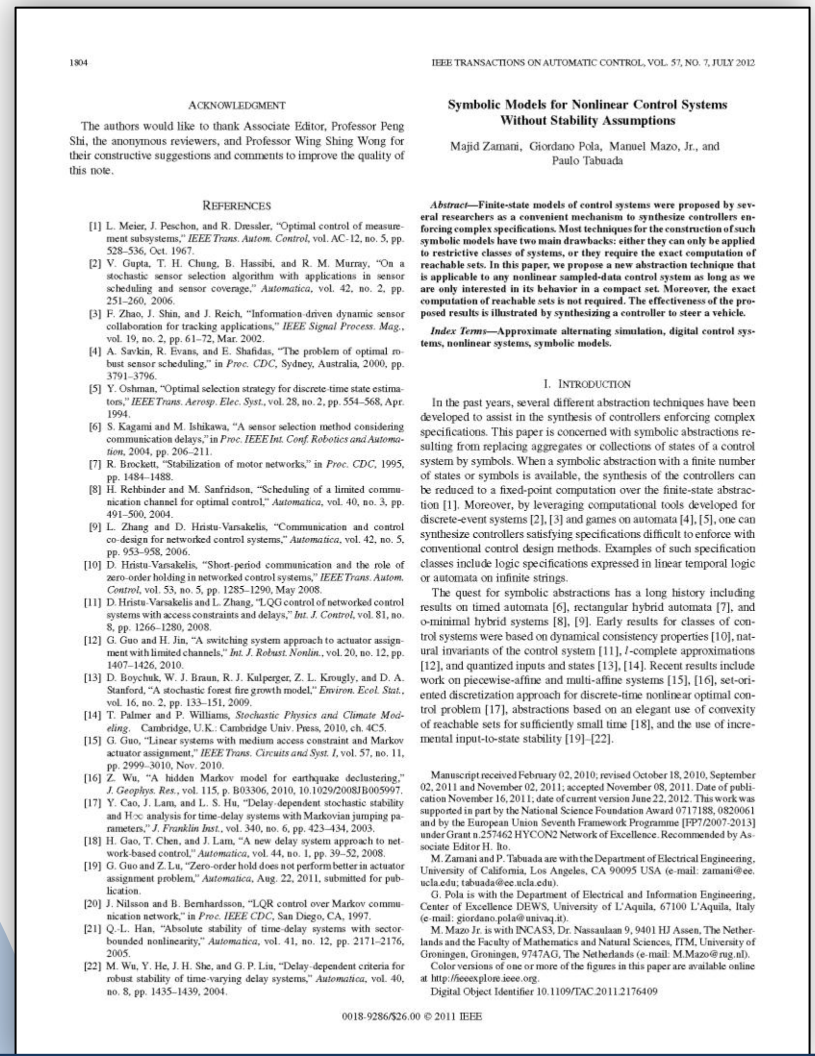
[Zamani, Pola, Mazo, Tabuada, IEEE-TAC-12]

A nonlinear control system dx/dt= f(x,u) is
<u>incrementally forward complete</u> (δ-FC)
if there exists a continuous function β
such that $\beta(s,.)$ is K_∞ for any real s and

$$||x(t,y,u) - x(t,z,u)|| \leq \beta(||y - z||,t)$$





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Symbolic Models for Nonlinear Control Systems Without Stability Assumptions

Majid Zamani, Giordano Pola, Manuel Mazo, Jr., and Paulo Tabuada

***Abstract*—Finite-state models of control systems were proposed by several researchers as a convenient mechanism to synthesize controllers enforcing complex specifications. Most techniques for the construction of such symbolic models have two main drawbacks: either they can only be applied to restrictive classes of systems, or they require the exact computation of reachable sets. In this paper, we propose a new abstraction technique that is applicable to any nonlinear sampled-data control system as long as we are only interested in its behavior in a compact set. Moreover, the exact computation of reachable sets is not required. The effectiveness of the proposed results is illustrated by synthesizing a controller to steer a vehicle.**

***Index Terms*—Approximate alternating simulation, digital control systems, nonlinear systems, symbolic models.**

I. INTRODUCTION

In the past years, several different abstraction techniques have been developed to assist in the synthesis of controllers enforcing complex specifications. This paper is concerned with symbolic abstractions resulting from replacing aggregates or collections of states of a control system by symbols. When a symbolic abstraction with a finite number of states or symbols is available, the synthesis of the controllers can be reduced to a fixed-point computation over the finite-state abstraction [1]. Moreover, by leveraging computational tools developed for discrete-event systems [2], [3] and games on automata [4], [5], one can synthesize controllers satisfying specifications difficult to enforce with conventional control design methods. Examples of such specification classes include logic specifications expressed in linear temporal logic or automata on infinite strings.

The quest for symbolic abstractions has a long history including results on timed automata [6], rectangular hybrid automata [7], and o-minimal hybrid systems [8], [9]. Early results for classes of control systems were based on dynamical consistency properties [10], natural invariants of the control system [11], *l*-complete approximations [12], and quantized inputs and states [13], [14]. Recent results include work on piecewise-affine and multi-affine systems [15], [16], set-oriented discretization approach for discrete-time nonlinear optimal control problem [17], abstractions based on an elegant use of convexity of reachable sets for sufficiently small time [18], and the use of incremental input-to-state stability [19]–[22].

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Symbolic models

Theorem [HSCC-2012]

For any δ-GAS nonlinear NCS Σ with compact state and input spaces and for any precision ε there exists a symbolic transition system $T^*(\Sigma)$ that is an ε-alternating approximate bisimulation of Σ and that can be effectively computed

Theorem [IEEE-CDC-2012]

For any δ-FC nonlinear NCS Σ with compact state and input spaces, for any precision ε, there exists a symbolic transition system $T^*(\Sigma)$ that is an ε-alternating approximate simulation of Σ and that can be effectively computed

Integrated Symbolic Design of Unstable Nonlinear Networked Control Systems

Alessandro Borri, Giordano Pola and Maria Domenica Di Benedetto

***Abstract*— The research area of Networked Control Systems (NCS) has been the topic of intensive study in the last decade. In this paper we give a contribution to this research line by addressing symbolic control design of (possibly unstable) nonlinear NCS with specifications expressed in terms of automata. We first derive symbolic models that are shown to approximate the given NCS in the sense of (alternating) approximate simulation. We then address symbolic control design with specifications expressed in terms of automata. We finally derive efficient algorithms for the synthesis of the proposed symbolic controllers which cope with the inherent computational complexity of the problem at hand. A computational complexity analysis of the proposed algorithms is discussed. An example is included which considers the remote symbolic control of a unicycle via an imperfect communication network with a motion planning type specification.**

I. INTRODUCTION

Networked Control Systems (NCS) are complex, heterogeneous, spatially distributed systems where physical processes interact with distributed computing units through non–ideal communication networks. The complexity and heterogeneity of such systems is given by the interaction of at least three components: a plant process that is often described by continuous dynamics, a controller implementing algorithms on microprocessors for the control of the plant, and a communication network conveying information between the plant and the controller which is often characterized by non-idealities such as variable sampling/transmission intervals, variable communication delays, quantization errors, packet dropouts, communication protocol and limited bandwidth. In the last decade, NCS have been the object of great interest in the research community and important results have been achieved, see e.g. [1] and the references therein. Most of the results on NCS mainly deals with stabilization problems under an imperfect communication network comprising a subset of the aforementioned communication non-idealities. The work in [2] instead, considers all the aforementioned communication non-idealities and proposes control algorithms for solving problems with complex specifications expressed in terms of automata. The main drawbacks of the results reported in [2] are:

(i) The plant in the NCS is supposed to be stable, which is quite restrictive in many application domains of interest.

(ii) The controllers proposed require a large computational complexity in their design.

The present work improves the results established in [2] in two directions:

(i') We extend out results to possibly unstable nonlinear control systems;

(ii') We design efficient algorithms which cope with the computational complexity of the approach in [2].

For (i') we generalize the results reported in [3] from nonlinear control systems to nonlinear *networked* control systems. For (ii') we generalize the control algorithms we proposed in [4] for stable nonlinear control systems to *unstable* nonlinear *networked* control systems. An example is included, which considers the remote symbolic control of a unicycle via an imperfect communication network with a motion-planning type specification.

The paper is organized as follows. Section 2 introduces the notation. In Section 3 we recall the class of nonlinear NCS considered in the paper. Section 4 reports some preliminary definitions that are employed in the sequel. Section 5 proposes symbolic models approximating NCS. In Section 6 we address symbolic control design and in Section 7 efficient control design algorithms. A simple example is included in Section 8. Finally, Section 9 offers some concluding remarks.

II. NOTATION

The symbols $\mathbb{N}$, $\mathbb{N}_0$, $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{R}_0^+$ denote the set of natural, nonnegative integer, integer, real, positive real, and nonnegative real numbers, respectively. Given a set A we denote $A^2 = A \times A$ and $A^{n+1} = A \times A^n$ for any $n \in \mathbb{N}$. Given an interval $[a, b] \subseteq \mathbb{R}$ with $a \leq b$ we denote by $[a; b]$ the set $[a, b] \cap \mathbb{N}$. We denote by $\lceil x \rceil = \min\{n \in \mathbb{Z} | n \geq x\}$ the ceiling of a real number x. Given a vector $x \in \mathbb{R}^n$ we denote by $\|x\|$ the infinity norm and by $\|x\|_2$ the Euclidean norm of x. Given $\mu \in \mathbb{R}^+$ and $A \subseteq \mathbb{R}^n$, we set $[A]_\mu = \mu\mathbb{Z}^n \cap A$; if $B = \bigcup_{i \in [1;N]} A^i$ then $[B]_\mu = \bigcup_{i \in [1;N]} ([A]_\mu)^i$. Consider a bounded set $A \subseteq \mathbb{R}^n$ with interior. Let $H = [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n]$ be the smallest hyperrectangle containing A and set $\hat{\mu}_A = \min_{i=1,2,\ldots,n}(b_i - a_i)$. It is readily seen that for any $\mu \leq \hat{\mu}_A$ and any $a \in A$ there always exists $b \in [A]_\mu$ such that $\|a - b\| \leq \mu$. Given $a \in A \subseteq \mathbb{R}^n$ and a precision $\mu \in \mathbb{R}^+$, the symbol $[a]_\mu$ denotes a vector in $\mu\mathbb{Z}^n$ such that $\|a - [a]_\mu\| \leq \mu/2$. Any vector $[a]_\mu$ with $a \in A$ can be encoded by a finite binary word of length $\lceil \log_2 |[A]_\mu| \rceil$. Given a pair of sets A and B and a relation $\mathcal{R} \subseteq A \times B$, the symbol $\mathcal{R}^{-1}$ denotes the inverse relation of $\mathcal{R}$, i.e. $\mathcal{R}^{-1} = \{(b, a) \in B \times A : (a, b) \in \mathcal{R}\}$. The cardinality of a finite set A is denoted by $|A|$.

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The research leading to these results has been partially supported by the Center of Excellence DEWS and received funding from the European Union Seventh Framework Programme [FP7/2007-2013] under grant agreement n.257462 HYCON2 Network of excellence.

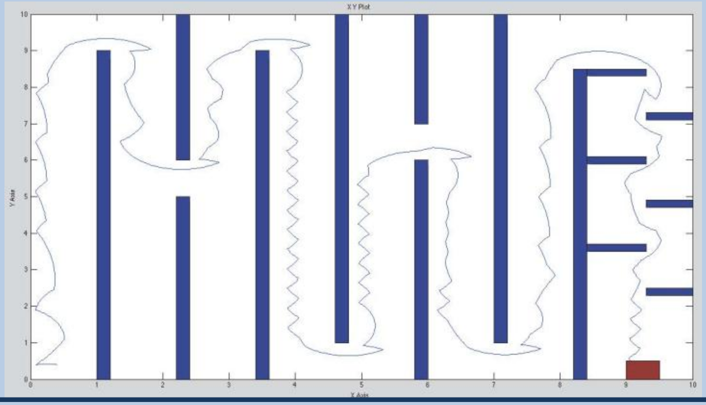
Symbolic control design

Class of specifications

Non-deterministic finite automata on infinite strings

Examples:

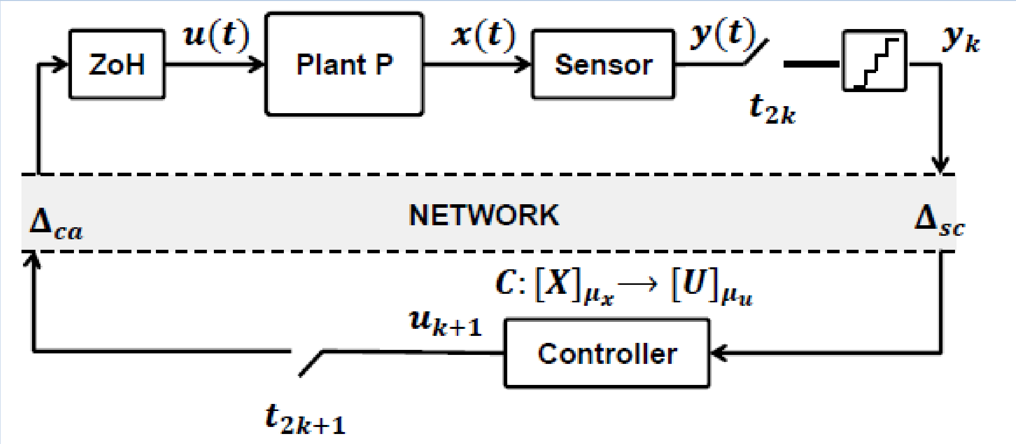
- Language specifications (e.g. robot motion planning)
- Synchronization specifications (e.g. starting from region A reach region B passing through region C in 1s)
- Obstacle avoidance (e.g. starting from region A, reach region B in finite time, while avoiding region C)
- Switching specifications (e.g. rotate clockwise in a certain region of the state space and rotate counter-clockwise in other regions of the state space)
- …



Symbolic control design

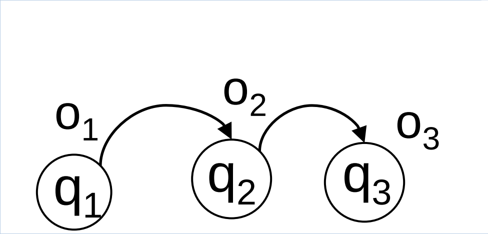
Problem formulation:

Given a NCS Σ, a specification LTS S and a desired precision ε > 0, find a symbolic controller that implements S (up to precision ε) robustly with respect to the non–idealities of the communication network and that is non-blocking when interacting with Σ



Networked Control System Σ

$\preccurlyeq_\varepsilon$

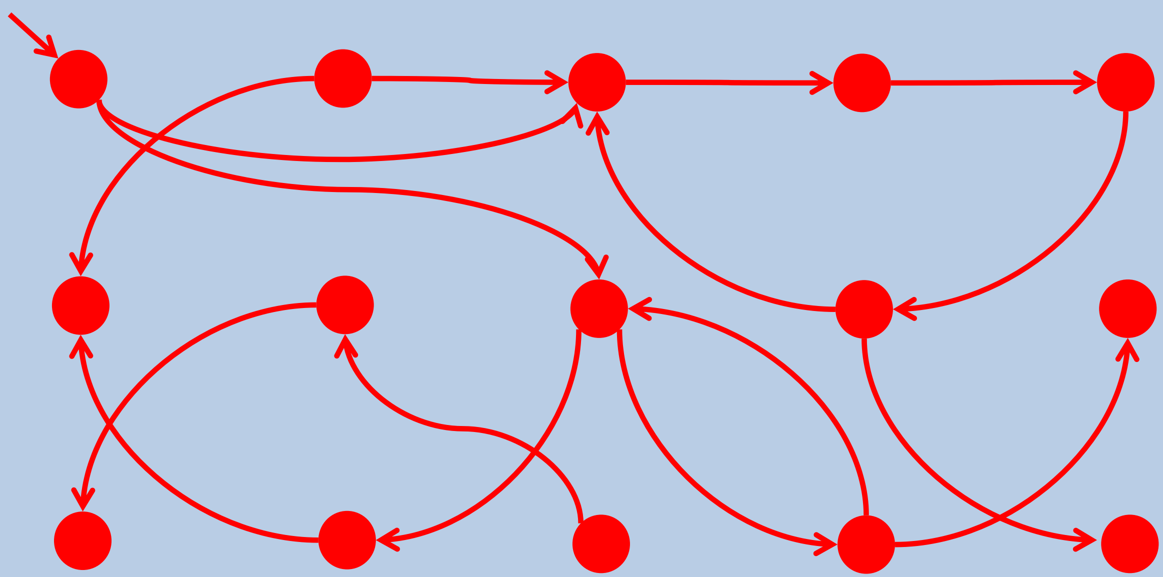


Specification LTS S

Symbolic control design

Synthesis through a three-step process:

1. Compute the symbolic model $T^*(\Sigma)$ of Σ
2. Compute the approximate parallel composition $C^* = T^*(\Sigma) \,||_{\mu x}\, S$
3. Compute the maximal robust non-blocking part $Nb(C^*)$ of C^*



Symbolic control design

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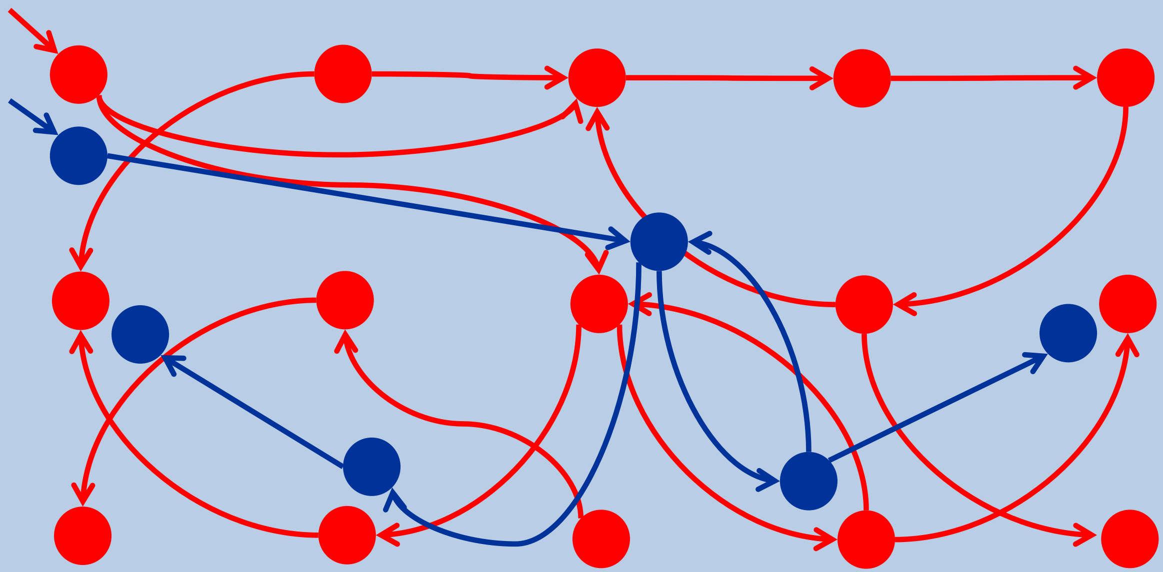

Symbolic control design

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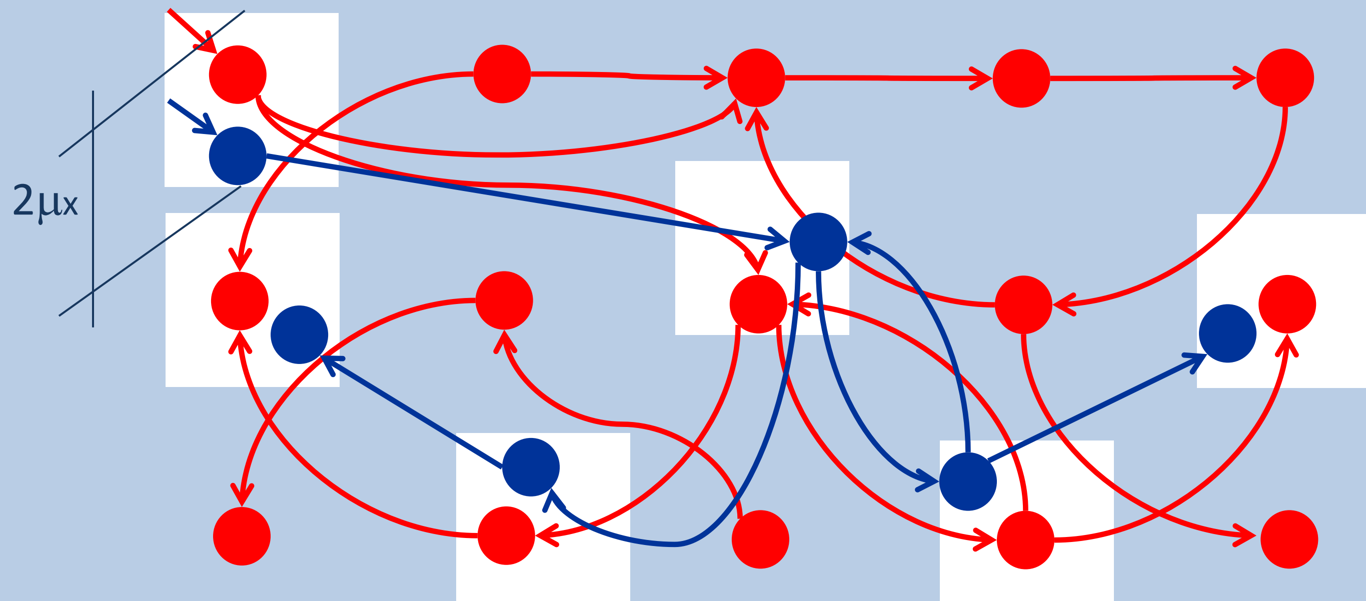

Symbolic control design

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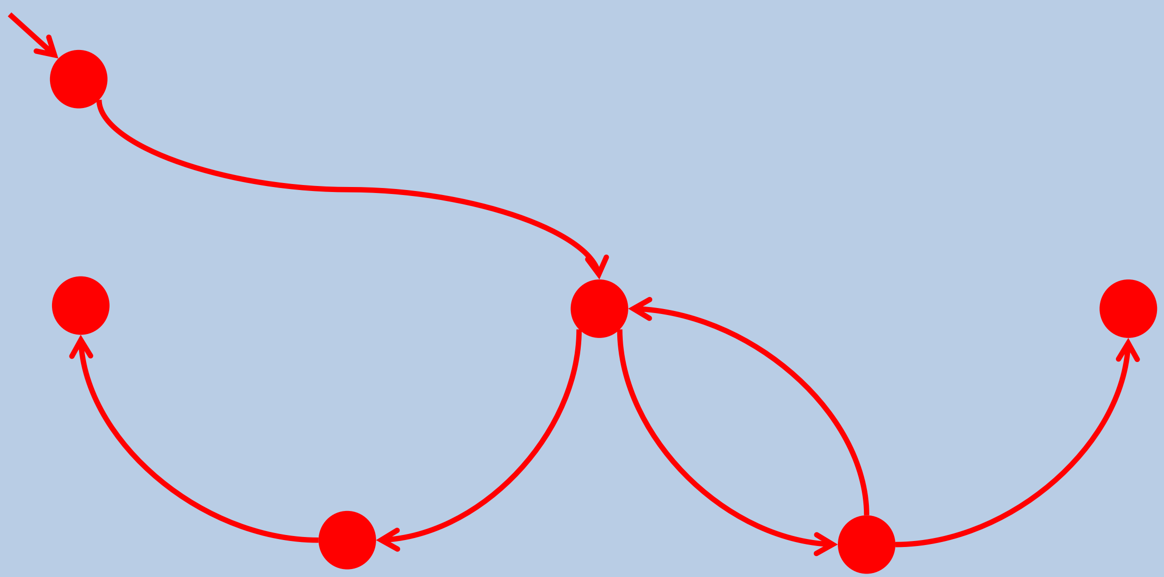

Symbolic control design

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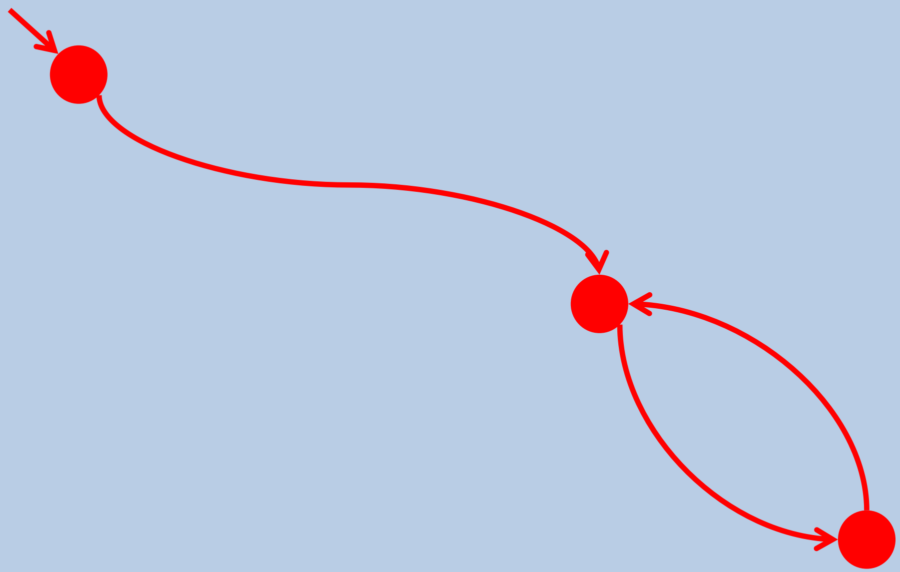

Symbolic control design

Synthesis through a three-step process:

1. Compute the symbolic model $T^*(\Sigma)$ of Σ
2. Compute the approximate parallel composition $C^* = T^*(\Sigma) \,||_{\mu x}\, S$
3. Compute the maximal robust non-blocking part $Nb(C^*)$ of C^*

Drawback

High computational complexity!

Symbolic control design

Synthesis through a three-step process:

1. Compute the symbolic model $T^*(\Sigma)$ of Σ
2. Compute the approximate parallel comp
3. Compute the maximal robust non-blocki

Drawback

High computational complexity!

Efficient on-the-fly (off-line) algorithms that integrate the synthesis of $Nb(C^*)$ with the construction of $T^*(\Sigma)$

[Pola, Borri, Di Benedetto, IEEE-TAC-2012]

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Integrated Design of Symbolic Controllers for Nonlinear Systems

Giordano Pola, *Member, IEEE*, Alessandro Borri, *Member, IEEE*, and Maria Domenica Di Benedetto, *Fellow, IEEE*

***Abstract*—Symbolic models of continuous and hybrid systems have been studied for a long time, because they provide a formal approach to solve control problems where software and hardware interact with the physical world. While being powerful, this approach often encounters some limitations in concrete applications, because of the large size of the symbolic models needed to be constructed. Inspired by on–the–fly techniques for verification and control of finite state machines, in this note we propose an algorithm that integrates the construction of the symbolic models with the design of the symbolic controllers. Computational complexity of the proposed algorithm is discussed and an illustrative example is included.**

***Index Terms*—Approximate bisimulation, digital control systems, nonlinear systems, on–the–fly design, symbolic models.**

I. INTRODUCTION

Symbolic models of continuous and hybrid systems have been studied for a long time, because they provide a formal approach to solve control problems where software and hardware interact with the physical world. Symbolic models are abstract descriptions of control systems in which a symbolic state corresponds to an aggregate of states. Several classes of dynamical and control systems that admit symbolic models were identified during the last few years, see, e.g., [1], [12] and the references therein. In particular, incrementally stable [2] nonlinear control systems were shown in [7], [10] to admit symbolic models. This last result has been further generalized to incrementally stable nonlinear switched systems in [6], incrementally stable nonlinear time–delay systems in [8], [9] and incrementally forward complete nonlinear control systems in [15]. The use of symbolic models for the control design of continuous and hybrid systems has been investigated in [11], [14]. As discussed in [12], this approach provides the designer with a systematic method to address a wide spectrum of novel specifications, that are difficult to enforce by means of conventional control design paradigms. Examples of such specifications include logic specifications expressed in terms of linear temporal logic formulae or automata on infinite strings. The use of these specifications has been shown to be relevant in the control design of important domains of application, including robot motion planning and systems biology (see, e.g., [14] and the references therein). While being powerful, this approach often encounters some limitations in concrete applications, because of the large size of the symbolic models needed to be constructed. In this note we propose one approach to cope with this drawback. We consider a symbolic control design problem for nonlinear control systems. Given a nonlinear control plant and a specification expressed in terms of a finite automaton on infinite strings, we face the problem of designing a symbolic controller that implements the specification with arbitrarily good accuracy. The symbolic controller is furthermore requested to avoid blocking behaviors, when interacting with the plant. This problem can be viewed as an approximate version of similarity games, as discussed in [12]. Related control design problems have been studied in [11] and [14]. The first contribution of this note lies in the derivation of an explicit solution to the control problem under study. *The symbolic controller is proven to be the non–blocking part [3] of the approximate parallel composition [12] between the specification automaton and the symbolic model of the plant*. The synthesis of such a controller requires the preliminary construction of the symbolic model of the plant, which is generally demanding from the computational complexity point of view. Inspired by the research line on on–the–fly verification and control of finite state machines (see e.g., [4], [13]), we give the second contribution of this note consisting in *an efficient algorithm that integrates the construction of the symbolic model of the plant with the design of the symbolic controller*. Computational complexity of the proposed algorithm is discussed and an illustrative example is included.

II. PRELIMINARY DEFINITIONS

Notation

The symbol $|A|$ denotes the cardinality of a finite set A. The identity map on a set A is denoted by 1_A. Given a relation $\mathcal{R} \subseteq A \times B$, the symbol $\mathcal{R}^{-1}$ denotes the inverse relation of $\mathcal{R}$, i.e., $\mathcal{R}^{-1} = \{(b,a) \in B \times A : (a,b) \in A \times B\}$. The symbols $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{R}_0^+$ denote the set of integer, real, positive real, and nonnegative real numbers, respectively. The symbol $\|x\|$ denotes the infinity norm of $x \in \mathbb{R}^n$. Given a measurable function $f : \mathbb{R}_0^+ \to \mathbb{R}^n$, the (essential) supremum of f is denoted by $\|f\|_\infty$. Given $x \in \mathbb{R}^n$ and $\varepsilon \in \mathbb{R}^+$, the symbols $\mathcal{B}_\varepsilon(x)$ and $\mathcal{B}_{[\varepsilon[}(x)$ denote the set $\{x \in \mathbb{R}^n \mid \|x\| \leq \varepsilon\}$ and the set $[-\varepsilon+x_1, x_1+\varepsilon[\times[-\varepsilon+x_2, x_2+\varepsilon[\times\cdots\times[-\varepsilon+x_n, x_n+\varepsilon[$, respectively. Given $\mu \in \mathbb{R}^+$ and $A \subseteq \mathbb{R}^n$, we denote by μA the set $\{b \in \mathbb{R}^n \mid \exists a \in A \text{ s.t. } b = \mu a\}$. For any $x \in \mathbb{R}^n$ and $\mu \in \mathbb{R}^+$ the symbol $[x]_\mu$ denotes the unique vector in $\mu\mathbb{Z}^n$ such that $x \in \mathcal{B}_{[\mu/2[}([x]_\mu)$.

A. Control Systems

In this note we consider the nonlinear control system

$$\Sigma : \begin{cases} \dot{x}(t) = f(x(t), u(t)), t \in \mathbb{R}_0^+, \\ x(0) \in X_0, \end{cases} \quad (1)$$

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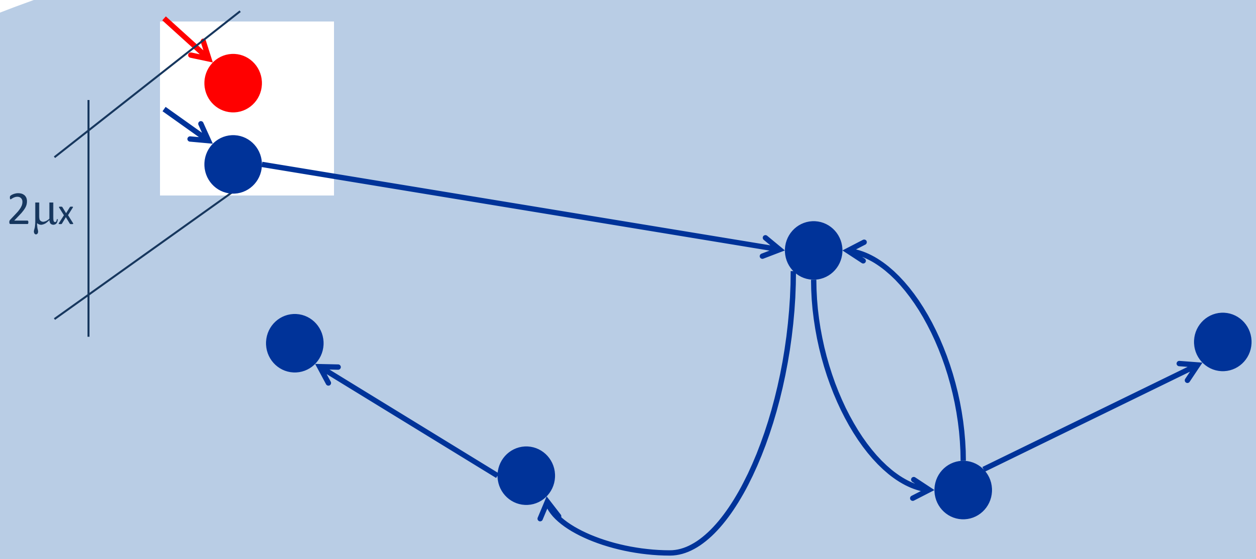
Symbolic control design

Synthesis through a three-step process:

1. Compute the symbolic model $T^*(\Sigma)$ of Σ
2. Compute the approximate parallel composition $C^* = T^*(\Sigma) \,||_{\mu x}\, S$
3. Compute the maximal robust non-blocking part $Nb(C^*)$ of C^*



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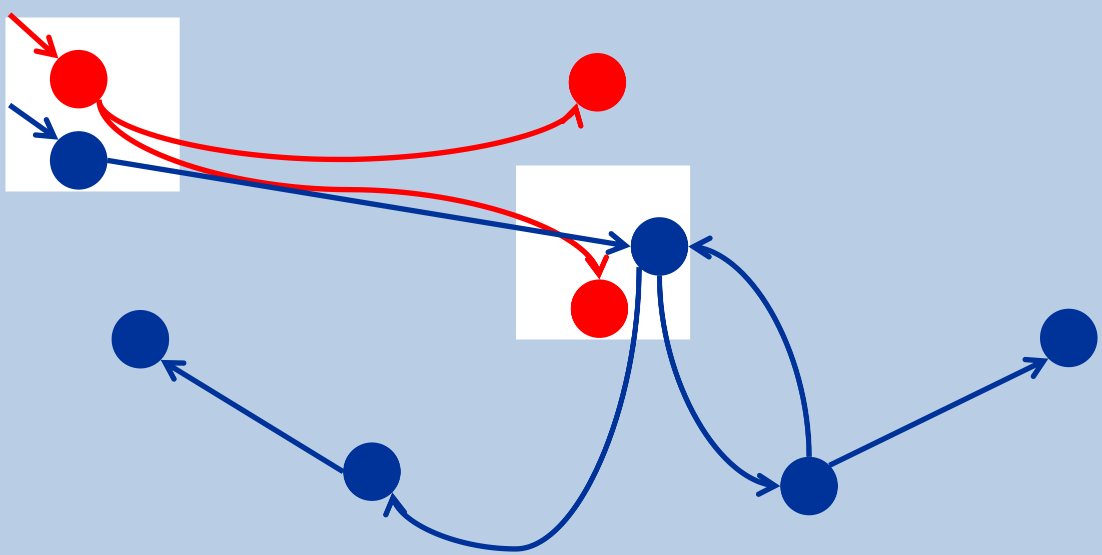

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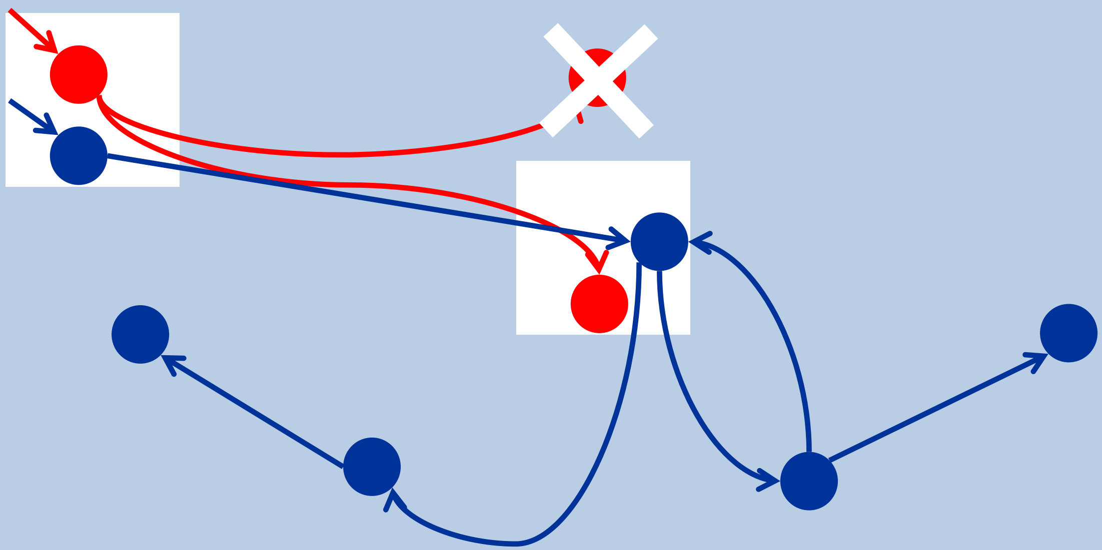

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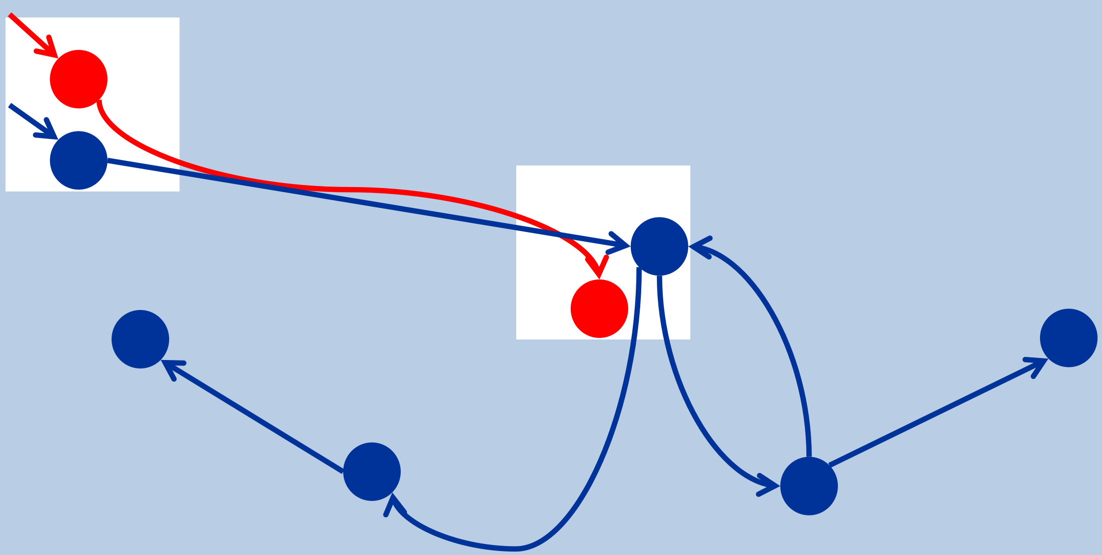

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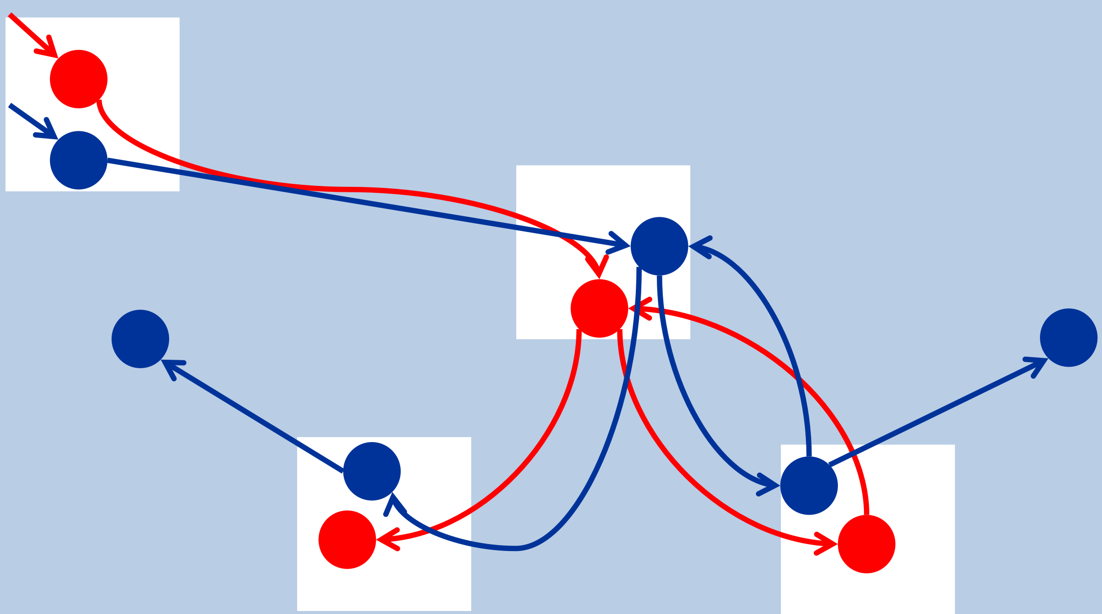

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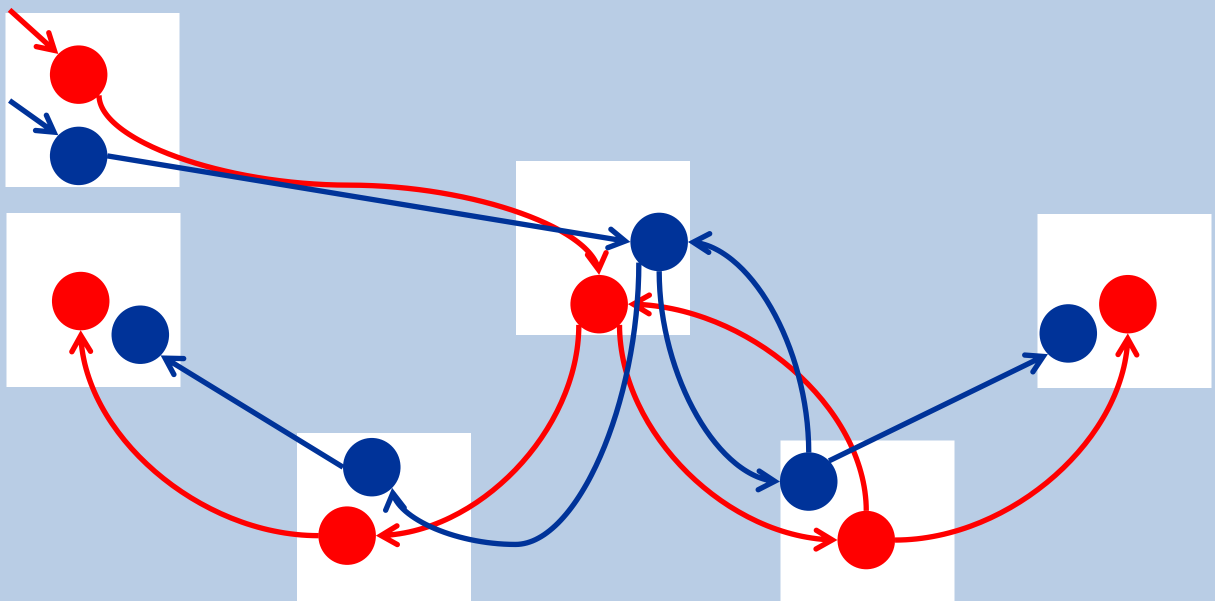

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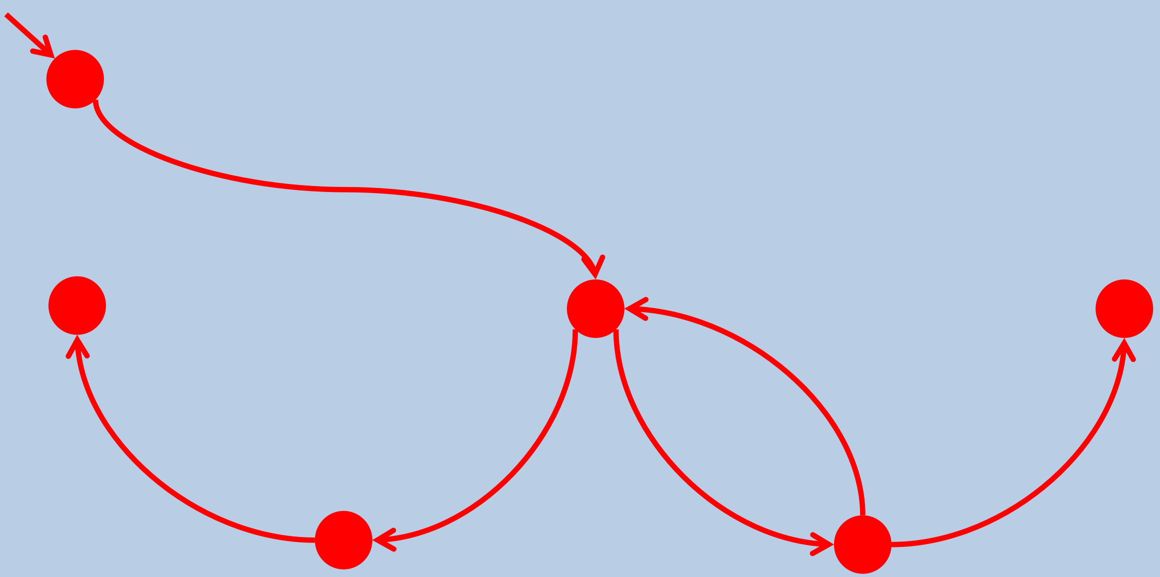

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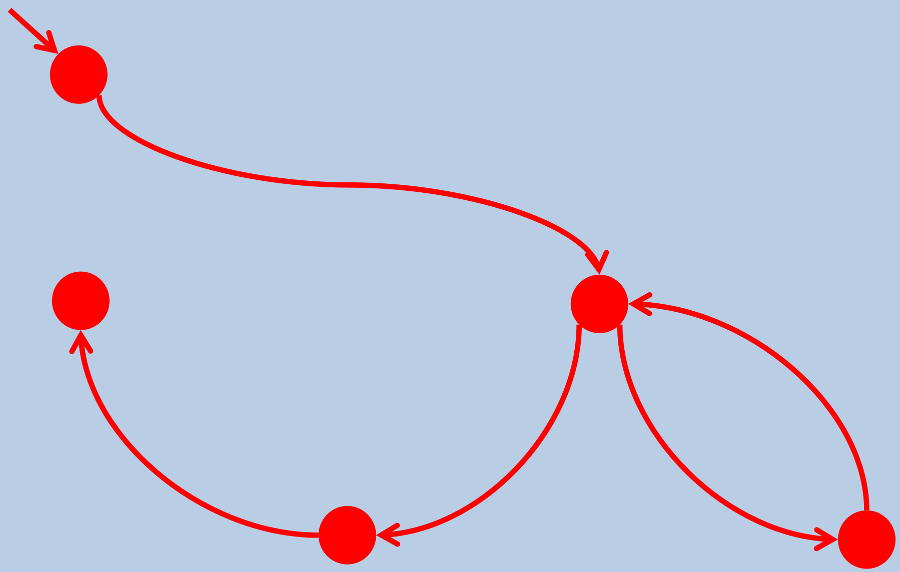

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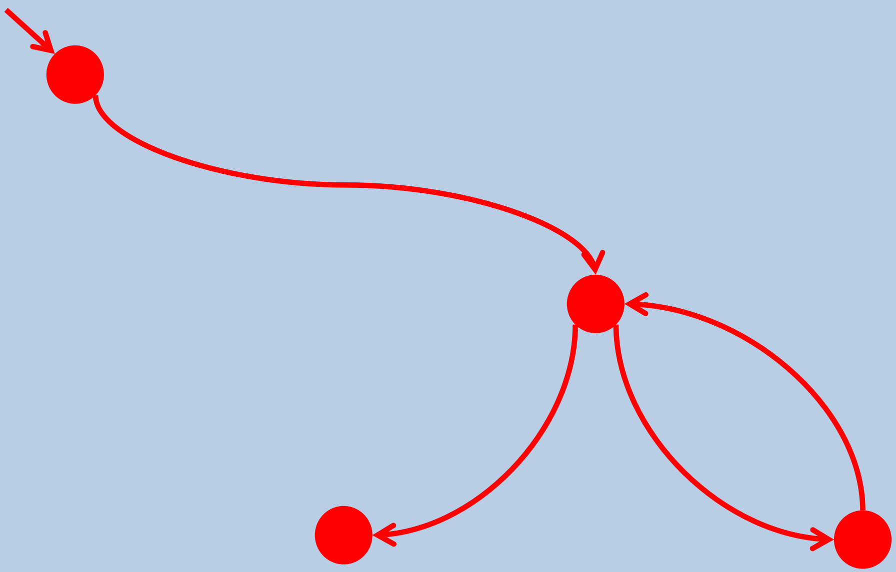

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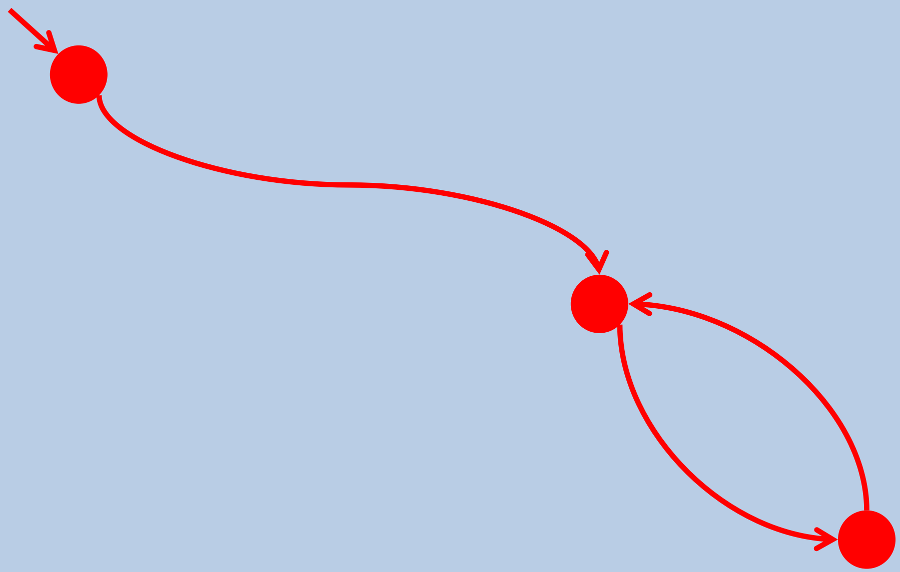

Conclusions

- Mathematical model of a rather general class of nonlinear NCS
- Symbolic models for stable and unstable nonlinear NCS
- Symbolic controllers for NCS to address a wide range of novel complex specifications
- Efficient control algorithms

Appendix

Appendix (1/3)

How to capture interaction between the symbolic model and the symbolic controller?

Approximate parallel composition

Def [Tabuada, IEEE-TAC-2008]

Given $T_1 = (Q_1, L_1, \longrightarrow_1, O_1, H_1)$ and $T_2 = (Q_2, L_2, \longrightarrow_2, O_2, H_2)$, with $O_1 = O_2$, and a precision $\theta > 0$, the approximate composition of T_1 and T_2 is the system

$$T_1 \,||_\theta\, T_2 = (Q, L, \longrightarrow, O, H)$$

where:

- $Q = \{(q_1, q_2) \in Q_1 \times Q_2 : d(H_1(q_1), H_2(q_2)) \leq \theta\}$
- $L = L_1 \times L_2$
- $(q_1, q_2) \xrightarrow{(l_1, l_2)} (q'_1, q'_2)$, if $q_1 \xrightarrow{l_1}_1 q'_1$ and $q_2 \xrightarrow{l_2}_2 q'_2$
- $O = O_1$
- $H(q_1, q_2) = H_1(q_1)$

Appendix (2/3)

How to deal with non-determinism of symbolic models?

Robust symbolic controllers

Def Given a LTS T = $(Q_s, Q_{s,0}, L_s, \longrightarrow_s, O_s, H_s)$, a symbolic controller

$$C = (Q_c, Q_{c,0}, L_c, \longrightarrow_c, O_c, H_c)$$

is said to be robust with respect to T with composition parameter θ > 0 if for any $l_s \in L_s$ and for each pair of transitions

$$q_s \xrightarrow{l_s}_s q'_s \quad \text{and} \quad q_s \xrightarrow{l_s}_s q''_s$$

in T, with $x'_s \neq x''_s$, the existence of a transition

$$(q_s, q_c) \xrightarrow{(l_s, l_c)} (q'_s, q'_c)$$

in $T||_\theta$ C for some q_c, q'_c implies the existence of a transition

$$(q_s, q_c) \xrightarrow{(l_s, l_c)} (q''_s, q''_c)$$

in $T ||_\theta$ C for some q''_c

Appendix (3/3)

Theorem 2 [HSCC-2012]

Suppose that the plant P in the NCS Σ is δ–GAS and satisfies the assumptions of Theorem 1. Then for any desired precision $\varepsilon > 0$, and any θ, $\mu_x > 0$ such that

$$\mu_x \leq min\left\{\gamma^{-1}\left(\left(1-e^{-\lambda\tau}\right)\underline{\alpha}(\theta)\right), \bar{\alpha}^{-1}(\underline{\alpha}(\theta)), \hat{\mu}_X\right\}$$

$$\mu_x + \theta \leq \varepsilon$$

the symbolic controller C* solves the control problem